Lambda sonar tomography for unbounded centersets

T.SCHUSTER
Department for Mechanical Engineering,
Helmut Schmidt University Hamburg,
Germany

In sonar tomography we want to recover a function f from its spherical means, where the center sets of the spheres are located at the hyperplane $\{x \in \mathbb{R}^{n+1} : x_{n+1} = 0\}$. Thus, we consider the operator

$$R_n f(z,r) = \frac{1}{|S^n|} \int_{S^n} f(z + r\xi, r\eta)\, \mathrm{d}S_n(\xi,\eta)\,,$$

where $(z,0)^\top \in \mathbb{R}^{n+1}$ and $r > 0$ is the center and the radius of the sphere, respectively, S^n denotes the unit sphere in $\mathbb{R}^{n+1}$ with surface measure $\mathrm{d}S_n$ and area $|S^n|$. The mathematical properties of R have been thoroughly investigated over the last two decades and inversion formulas are known. Since the center set is unbounded, R can not be formulated as bounded operator neither on L_2-spaces, nor on Sobolev spaces of negative order, but only on appropriate distribution spaces. Numerical inversion of R_n is very challenging. The inversion formulas for R_n contain non-local operators as the Hilbert transform. If we apply the method of approximate inverse, we have to compute the reconstruction kernels numerically which is too time consuming for dimensions higher than two. In the talk we discuss two lambda operators for the three-dimensional case ($n = 2$)

$$\begin{aligned}
\Lambda_1 f &= R_2^*\Big(\varphi_\varepsilon(z)\,(\frac{\partial^3}{\partial r^3} + \mu)\, R_2 f(z,r)\Big)(x)\,,\\
\Lambda_2 f &= R_2^*\Big(\frac{\varphi_\varepsilon(z)}{r}\,(\frac{\partial^3}{\partial r^3} + \mu)\, R_2 f(z,r)\Big)(x)\,.
\end{aligned}$$

Here, φ_ε denotes a smooth function having the compact support $[-\varepsilon,\varepsilon]^2$ turning Λ_i into a local, elliptic pseudodifferential operator. Obviously $\Lambda_i f \neq f$, but $\Lambda_i f$ has the same singular support as f. Furthermore, reconstruction kernels for Λ_i can be computed analytically. After a short discussion of Λ_1 and Λ_2 we outline a numerically stable realization of $\Lambda_i f$ with the help of the method of approximate inverse and show some first numerical experiments.

The contents of the talk represent joined work with E.T. Quinto (Tufts University, Medford, USA) and A. Rieder (TU Karlsruhe, Germany).