A rigorously justified algebraic preconditioner for diffusion problems with high-contrast coefficients

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ABSTRACT

In this paper we present a new algebraically motivated preconditioner for finite element approximations of elliptic PDEs with high-contrast coefficients. The construction of the preconditioner consists of two phases. The first phase is an algebraic one which identifies groups of freedoms corresponding to "high" and "low" permeability regions. These regions may be of arbitrary geometry. This partition of the freedoms yields a corresponding blocking of the stiffness matrix and hence a formula for its exact inverse involving the inverses of both the high permeability block and its Schur complement with respect to the original matrix. The second phase involves an efficient multigrid approximation of this exact inverse. By applying singular perturbation theory to each of the sub-blocks (where the small parameter is the reciprocal of the contrast of the coefficients), we show that each of them can be approximately inverted efficiently using geometric (or algebraic) multigrid methods, and that this inversion process is robust with respect to both the contrast and the mesh size. The result is a multigrid method for high contrast problems which is provably optimal to both contrast and mesh size. While a similar performance is also achieved in practice by algebraic multigrid methods alone, this performance is still without theoretical justification. Since the first phase of our method is comparable to the process of identifying weak and strong connections in conventional algebraic multigrid algorithms, our theory provides to some extent a step towards such a theoretical justification. We demonstrate the advantageous properties of our preconditioner using experiments on model high contrast problems and also on more realistic test problems found in the petroleum engineering literature.