FETI methods for multiscale elliptic PDEs

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ABSTRACT

In this work we consider a Poisson-type equation in two and three dimensions with a varying coefficient, i. e.,

$$-\nabla \cdot [\alpha(x) \nabla u(x)] = f(x) \quad \text{for } x \in \Omega\,,$$

and with some Dirichlet and/or Neumann boundary conditions on $\partial\Omega$. We are interested in solvers of the underlying finite element system which are in a certain sense robust with respect to the variation in $\alpha(\cdot)$. A great success has been made with FETI-type domain decomposition methods: If the domain Ω can be composed into regular subdomains Ω_i where $\alpha(\cdot)$ is constant or slightly varying on each of the subdomain, one can construct robust preconditioners to be used in the iterative solution of the discretized PDE. It was shown that the condition number of the preconditioned system behaves like

$$\mathcal{O}(\max_i \, (1 + \log(H_i/h_i))^2)\,,$$

where H_i denotes the subdomain diameter and h_i the subdomain mesh size, and the estimate is robust with respect to the values of α, i. e., robust in the jumps across subdomain interfaces.

In the present work, we would like to further generalize these standard results on FETI methods. We have two applications in mind: First, there is the case of coefficient jumps not aligned with the subdomain interfaces. Secondly, the coefficient can be smooth but highly varying within one subdomain. The latter situation appears when considering Newton-type problems, e. g., solving nonlinear magnetic field problems. We propose modifications of the standard FETI preconditioners which depend only very mildly on the variation of the coefficients near the interfaces.

If we assume that we have arbitrary variationof the coefficient in a part of the subdomain which is separated from the boundary by a distance $\eta_i > 0$ and smooth variation of at most α_i^* in the remainder of the domain, we can even give a rigorous analysis. In this case we can show that the condition number can be bounded by

$$\mathcal{O}\Big(\max_i \alpha_i^* \Big(\frac{H_i}{\eta_i}\Big)^d (1 + \log(H_i/h_i))^2 \Big)\,,$$

where d is the spatial dimension.

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