A multiscale approach to ion transport through channels

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In our paper, we consider the ion transport through a system of ion channels embedded into the cell membrane, separating the intracellular and extracellular space. Since the thickness of the membranes is very small compared with the dimension of the cell, systems including membranes have to be treated as multiscale systems.

Starting from a microscopic description based on the Poisson-Nernst-Planck equations, and performing an asymptotic analysis with respect to the small parameter given by the ratio between the thickness of the membrane and the dimension of the cell, we derive effective laws for the ion transport through membranes.

On the macroscopic level the membrane is treated as an interface between the intracellular and the extracellular space. The effective model consists in the Poisson-Nernst-Planck equations on both sides of the membrane together with appropriate jump conditions for the ion concentrations and the electric potential across the cell membrane. The jump conditions take into account the effects coming from the geometry of the membrane channels and their physiology.