Multilevel methods for non-conforming finite element approximations of elliptic problems with large jumps in the coefficients

J. Kraus[†] (joint work with I. Georgiev[*] and S. Margenov[‡])

[†]Johann Radon Institute for Computational and Applied Mathematics,
Austrian Academy of Sciences

[*‡]Institute for Parallel Processing,
Bulgarian Academy of Sciences

In this talk we discuss multilevel preconditioners in the framework of generalized hierarchical bases (GHB). In particular, we focus on non-conforming finite element (FE) approximations of second-order elliptic partial differential equations (PDE) using so-called Rannacher-Turek rotated bilinear (in two space dimensions) or trilinear (in three space dimensions) finite elements. We present different two-level splittings of the underlying FE spaces and analyze the angle between the coarse space and its complementary space. The GHB transformations that induce the splittings can be applied recursively, which allows us to construct optimal order multilevel methods of multiplicative and additive type. The presented numerical experiments address 3D problems with large jumps of the PDE coefficients that can only be resolved on the fine mesh.

At the end of the talk we give some perspectives, and discuss new developments of multilevel methods for discontinuous Galerkin (DG) approximations. In this case, an alternative construction of GHB is a key for achieving the desired robustness of the method with respect to arbitrary jumps of the PDE coefficients and thus DG techniques gain in attractiveness in microstructural FE analysis. This, however, is work in progress.