Domain decomposition for multiscale elliptic PDEs

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In this talk we will discuss domain decomposition preconditioning for linear systems arising from finite element approximation of symmetric elliptic problems with highly variable coefficients. The work is motivated by problems arising in the computation of flow through heterogeneous media in both deterministic and random cases, where coefficients may vary very rapidly over complicated structures.

Our overall aim is to solve such problems in a time close to the time required for solving a standard smooth Poisson problem.

We will give condition number bounds which demonstrate how the subdomains and coarse space basis functions should be designed in order for the methods to be robust to both the heterogeneity of the media and the mesh parameters. In particular, we give a rigorous explanation why low energy coarse space basis functions lead to robust methods. This naturally suggests that multiscale finite elements could provide good coarse spaces and also leads to a rigorous explanation of the success of certain algebraic preconditioners.

The results are illustrated by numerical examples on deterministic and random problems.

This is joint work with R. Scheichl.