MatCont: A numerical bifurcation tool for modeling.

We discuss the software packages CL_MATCONT and its GUI version MATCONT which are Matlab toolboxes for the study of dynamical systems. They were and are being developed in collaboration with Yuri A. Kuznetsov (Utrecht,NL) and several PhD students at Ghent and Utrecht, in particular Annick Dhooge, Bart Sautois, Hil Meijer and Reza Khoshsiar Ghaziani. The current versions are available at:
`http://matcont.ugent.be`.

Continuous dynamical systems (ODEs) have the generic form

$$\frac{dx}{dt} = f(x, \alpha),$$

where $x, f(x, \alpha) \in^n$ (the state space) and $\alpha \in^p$ (the parameter space); f is a nonlinear function. Such systems arise in many applications and notably in Systems Biology where the special feature is that they often have many parameters whose values are hard to obtain experimentally.

The most important types of solutions are equilibria ($x(t) = x_0$ for all t and a fixed x_0), periodic orbits ($x(t + T) = x(t)$ for all t and a fixed period T) and homoclinic orbits ($\lim_{x\to\pm\infty} = x_0$ for a fixed x_0).

The behaviour of dynamical systems often changes dramatically if the parameter changes. Codimension 1 and 2 bifurcations are fairly well understood though the local behaviour of dynamical systems near codimension 2 points can be very complicated.

The numerical study of dynamical systems consists of many tasks. The fundamental, relatively simple tool is numerical continuation, i.e. the study of solutions of a given type under parameter variation. The second is the detection and location of encountered bifurcations. The third is the classification of the bifurcation by the computation of the so-called normal forms. The coefficients that appear in these normal forms determine the existence and stability of the new objects that are born in the bifurcation points. Unfortunately, these normal form coefficients depend on higher order derivatives of f (up to order 5 for codimension 2). Next, it is necessary to provide branch-switching routines to compute the new branches that are born in bifurcation points.

We discuss the functionalities of the software, its applicability in the study of practical problems and challenges for future improvements and extensions.