Self-similar solutions of a parabolic system of chemotaxis

We consider solutions of a parabolic system of chemotaxis

$$\begin{aligned} \frac{\partial u}{\partial t} &= \nabla \cdot (\nabla u - u \nabla v), \quad x \in \mathbb{R}^2,\ t > 0, \\ \tau \frac{\partial v}{\partial t} &= \Delta v + u, \quad x \in \mathbb{R}^2,\ t > 0, \end{aligned}$$

with $\tau \geq 0$, generalizing the parabolic-elliptic Patlak-Keller-Segel system with $\tau = 0$.

Existence of self-similar solutions is studied for a control parameter $M \equiv \int_{\mathbb{R}^2} u(x,t)\, dx$ in an interval $[0, M_\tau)$. The role of self-similar solutions in a description of the long time behavior of general solutions of the Cauchy problem is also investigated.