Numerical Solution of Optimization Problems in Finance

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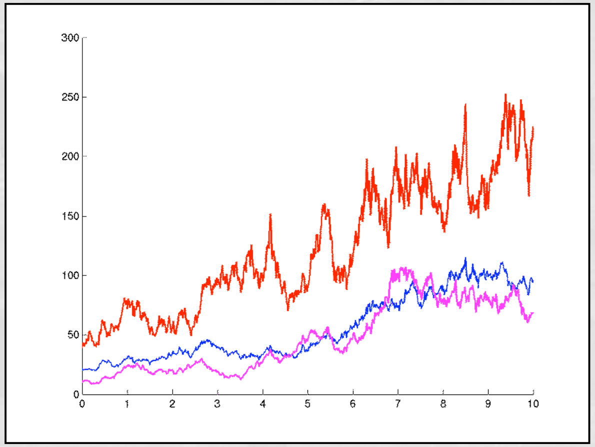
Current Research Interests

- Optimization in finance
- Robust optimization, hedging
- Nonlinear programming
- Parameter Identification
- Stochastic Programming
- Stochastic Differential Equations



Current industry projects:

- Robust static super-replication of barrier options
- Calibration of stochastic volatility models

Project partners: HypoVereinsbank UniCredit

Robust Static Hedging of Barrier Options

A bank sells a claim with payoff C¸0 at time T.

Time	Today: t=0	Future: t=T
Cash flow	Price of C	-C(ω)

Goal of the Seller:
Find a cheap portfolio of alternative financial instruments such that no future losses can occur.

⇒ This leads to a robust optimization problem of the form

$$\min_{\alpha\in\mathbb{R}^n} \sum_{i=1}^{n} \alpha_i C^i(0, S_0, Y_0, \vec{p}_0)$$

s.t.

$$\sum_{i:T_i\geq t} \alpha_i C^i(t, D, y, \vec{p}) \geq 0 \quad \forall\ (t, y) \in [0, T] \times \bar{Y},\ \forall \vec{p} \in P \subset \mathbb{R}^k$$

$$\sum_{i:T_i=T} \alpha_i (s - K_i)^+ \geq (s - K)^+ \quad \forall\ s \in [0, D]$$

$$C_t^i + rxC_x^i + \alpha(y, \vec{p})C_y^i + \frac{1}{2}x^2\sigma(y, \vec{p})^2 C_{xx}^i + \frac{1}{2}\beta(y, \vec{p})^2 C_{yy}^i + \rho x\sigma(y, \vec{p})\beta(y, \vec{p})C_{xy}^i = rC^i$$

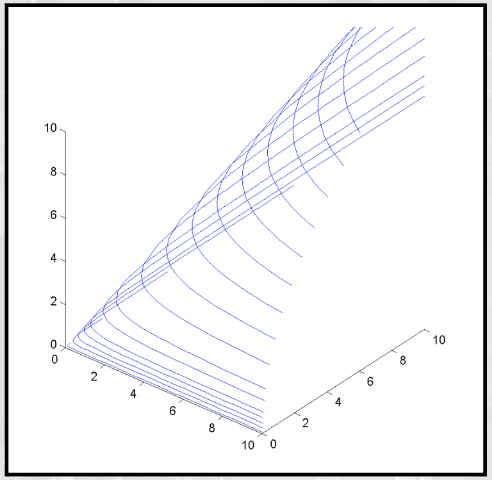
Calibration of Stochastic Volatility Models

Task: Calibrate stochastic volatility models to given market data

⇒ As financial market data changes very quickly, fast numerical algorithms are required

Combining several optimization techniques leads to an efficient algorithm:

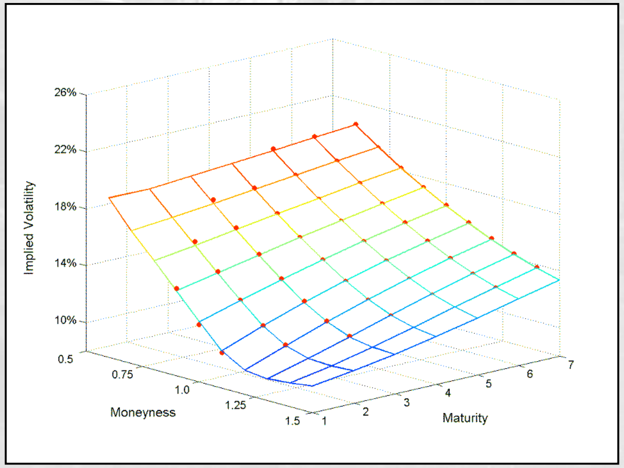
- Feasibility perturbed SQP
- Gauss-Newton approximation of the Hessian
- Trust region step size control
- Analytically derived projections
- Semidefinite programming for projections including box constraints



Possible Colaborations

Parameter identification problems in Finance

- Existence and Uniqueness of local / global solutions
- Regularization techniques



Sample Average Approximation Methods (SAA)

- Gather numerical experience by incorporating Quasi Monte Carlo Methods in a nested SAA context
- Convergence theory in a trust region setting