Parameter Identification in Piezoelectricity

DFG Junior Research Group

Inverse Problems in Piezoelectricity

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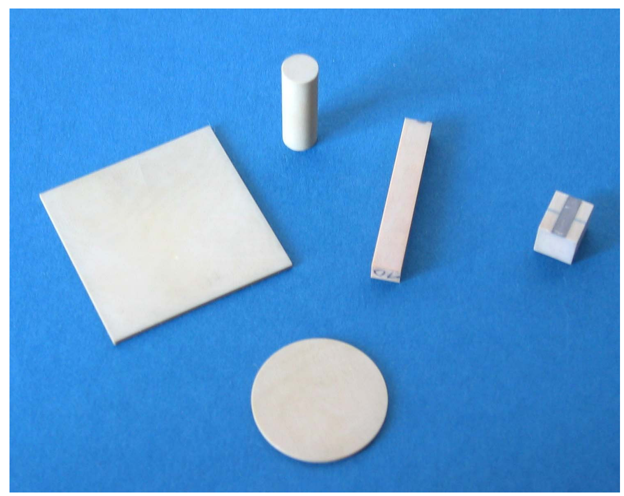

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Research: Inverse Problems

- Piezoelectric equations

$$\begin{aligned} \rho\frac{\partial^2\vec{u}}{\partial t^2} - DIV\left(c^E DIV^T\vec{u} + e^T grad\phi\right) &= 0 \in \Omega \\ -div\left(e DIV^T\vec{u} - \varepsilon^S grad\phi\right) &= 0 \in \Omega \end{aligned}$$

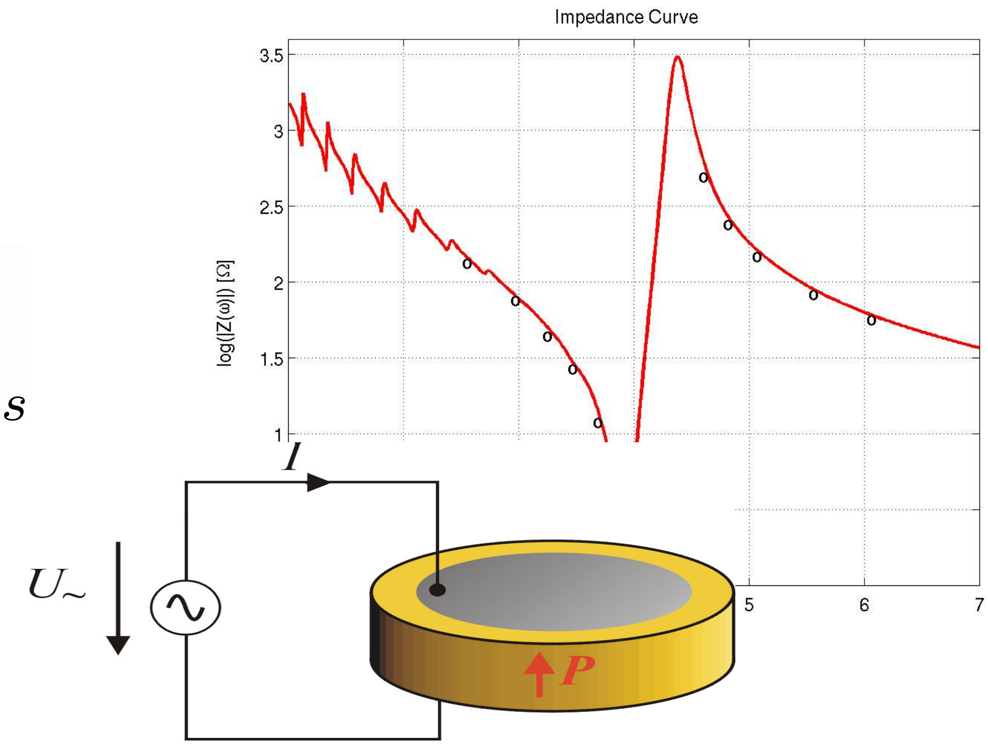
- Discretization by FEM

- Inverse Problem

$$\hat{\mathbf{F}} : X^{par} \rightarrow Y^{meas}$$

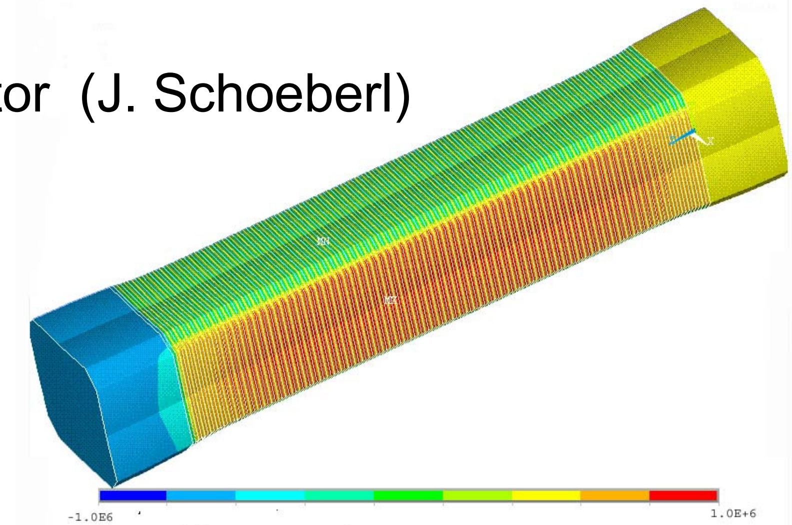
$$\hat{\mathbf{F}}(\mathbf{c}^E, \mathbf{e}, \varepsilon^S) = \hat{q}^e_{meas}$$

- Multilevel methods for nonlinear problems



Open Tasks – Discussions at RICAM

- Homogenization for stack actuator (J. Schoeberl)



- Optimal experiment design (V. Schulz)

- Nonlinear piezoelectric equations:

$$
\begin{aligned}
c^E &= c^E(|\mathcal{B}\vec{d}|) \\
e &= e(|\mathcal{B}\vec{d}|, |\mathrm{grad}\phi|) \\
\varepsilon^S &= \varepsilon^S(|\mathrm{grad}\phi|)
\end{aligned}
$$

- Identifiability results -> (G. Nakamura)

- Adaptivity -> (B. Vexler, H. Egger)

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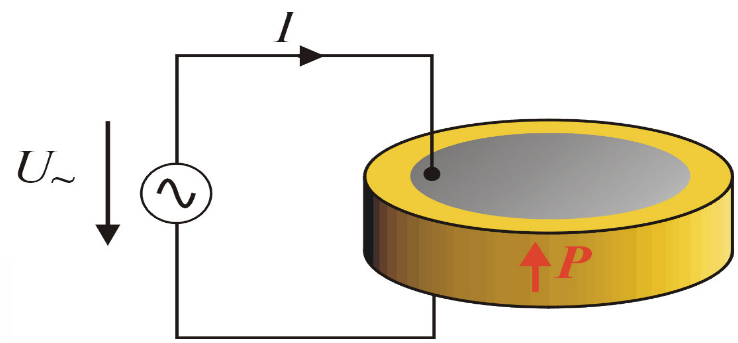
Any Questions ?

Piezoelectric PDEs (transient)

$$\rho\frac{\partial^2\vec{u}}{\partial t^2} - DIV\left(c^E DIV^T\vec{u} + e^T grad\phi\right) = 0 \in \Omega$$
$$-div\left(eDIV^T\vec{u} - \varepsilon^S grad\phi\right) = 0 \in \Omega$$



Boundary conditions:

$$N^T\sigma = 0 \quad \text{on}\,\partial\Omega$$
$$\phi = 0 \quad \text{on}\,\Gamma_g\text{...grounded electrode}$$
$$\phi = \phi^e \quad \text{on}\,\Gamma_e\text{...loaded electrode}$$
$$\vec{D}\cdot\vec{N} = 0 \quad \text{on}\,\partial\Omega\setminus(\Gamma_g\cup\Gamma_e)$$

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Piezoelectric Material Law

(6 mm crystal class)

$$
\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \\ D_1 \\ D_2 \\ D_3 \end{pmatrix}
=
\begin{pmatrix}
\mathbf{c}_{11}^E & \mathbf{c}_{12}^E & \mathbf{c}_{13}^E & \cdot & \cdot & \cdot & \cdot & \cdot & -\mathbf{e}_{13} \\
\mathbf{c}_{12}^E & \mathbf{c}_{11}^E & \mathbf{c}_{13}^E & \cdot & \cdot & \cdot & \cdot & \cdot & -\mathbf{e}_{13} \\
\mathbf{c}_{13}^E & \mathbf{c}_{13}^E & \mathbf{c}_{33}^E & \cdot & \cdot & \cdot & \cdot & \cdot & -\mathbf{e}_{33} \\
\cdot & \cdot & \cdot & \mathbf{c}_{44}^E & \cdot & \cdot & \cdot & -\mathbf{e}_{15} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \mathbf{c}_{44}^E & \cdot & -\mathbf{e}_{15} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \mathbf{c}_{66}^E & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \mathbf{e}_{15} & \cdot & \varepsilon_{11}^S & \cdot & \cdot \\
\cdot & \cdot & \cdot & \mathbf{e}_{15} & \cdot & \cdot & \cdot & \varepsilon_{11}^S & \cdot \\
\mathbf{e}_{13} & \mathbf{e}_{13} & \mathbf{e}_{33} & \cdot & \cdot & \cdot & \cdot & \cdot & \varepsilon_{33}^S
\end{pmatrix}
\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ E_1 \\ E_2 \\ E_3 \end{pmatrix}
$$

where $\mathbf{c}_{66}^E = \frac{1}{2}(\mathbf{c}_{11}^E + \mathbf{c}_{12}^E)$

elasticity [N/m²],

piezoelectric coupling [(C/m²)],

permittivity [F/m]

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Piezoelectric Effect

$$\vec{\sigma} = \mathbf{c}^E \vec{S} - \mathbf{e}^T \vec{E}$$
$$\vec{D} = \mathbf{e} \vec{S} + \varepsilon^S \vec{E}$$

$\vec{\sigma}$	...	mechanical stress
$\vec{S} = DIV \vec{u}$	...	mechanical strain
$\vec{E} = -grad \phi$	...	electric field
$\vec{D}$	...	dielectric displacement
$\vec{u}$	...	mechanical displacement
ϕ	...	electric potential

+ Newton's law: $\nabla \cdot \vec{\sigma} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}$

+ Gauss' law: $\nabla \cdot \vec{D} = 0$

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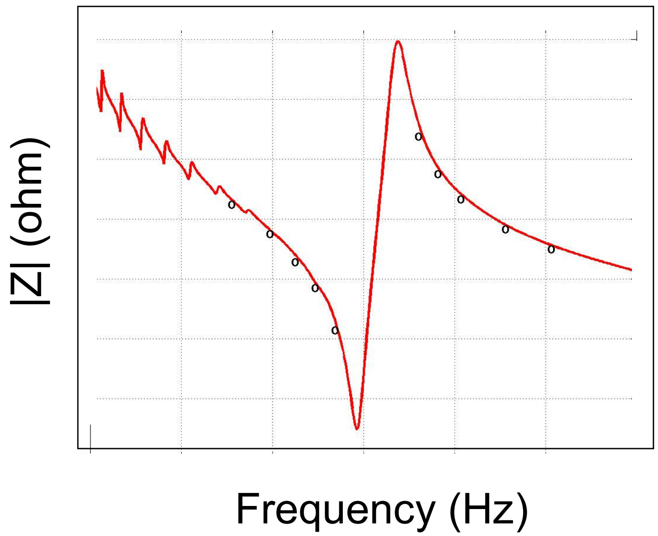
Inverse Problem – Identification by Simulation of Piezoelectric PDEs



Find material tensors $\mathbf{c}^E$, $\mathbf{e}$, ε^S *from impedance measurements for different frequencies* ω

$$Z(\omega) = \frac{\hat{\phi}^e(\omega)}{j\omega\hat{q}^e(\omega)}$$

$$\hat{q}^e = \int_{\Gamma_e} \vec{n}\left(\mathbf{e}DIV^T\hat{\vec{u}} - \varepsilon^S \mathrm{grad}\hat{\phi}\right) \mathrm{d}\Gamma_e$$

Z – impedance, ϕ^e – impressed voltage, $\hat{q}^e$ – surface charge

- Nonlinear operator equation $\hat{\mathbf{F}}(\mathbf{c}^E, \mathbf{e}, \varepsilon^S) = \hat{q}^e_{meas}$
- Forward operator $\hat{\mathbf{F}}$ involves set of PDE solution

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Inverse Problem – Solution/ Regularization by Inexact Newton Methods

Choose $\mathbf{p}^0 = (\mathbf{c}^E, \mathbf{e}, \varepsilon^S)^0$;
set $k = 0$;
while $\|\hat{y} - \hat{\mathbf{F}}(p^k)\| \geq \tau\delta$ do
    set $s_0^k = 0$;
    while $||\hat{y} - \hat{\mathbf{F}}(p^k) - \hat{\mathbf{F}}'(p^k)[s_n^k]|| \geq \eta_k ||\hat{y} - \hat{\mathbf{F}}(p^k)||$ do
        $s_n^k = \Phi\left(\hat{\mathbf{F}}'(p^k), \hat{y} - \hat{\mathbf{F}}(p^k), s_{n-1}^k\right)$;
        $n++$;
    $\mathbf{p}^{k+1} = \mathbf{p}^k + s_n^k$;
    $k++$;

δ - data noise, $\tau > 1$, η_k - tolerance factor
Choices for $\Phi(...)$:
Landweber's iteration, ν-methods, CG

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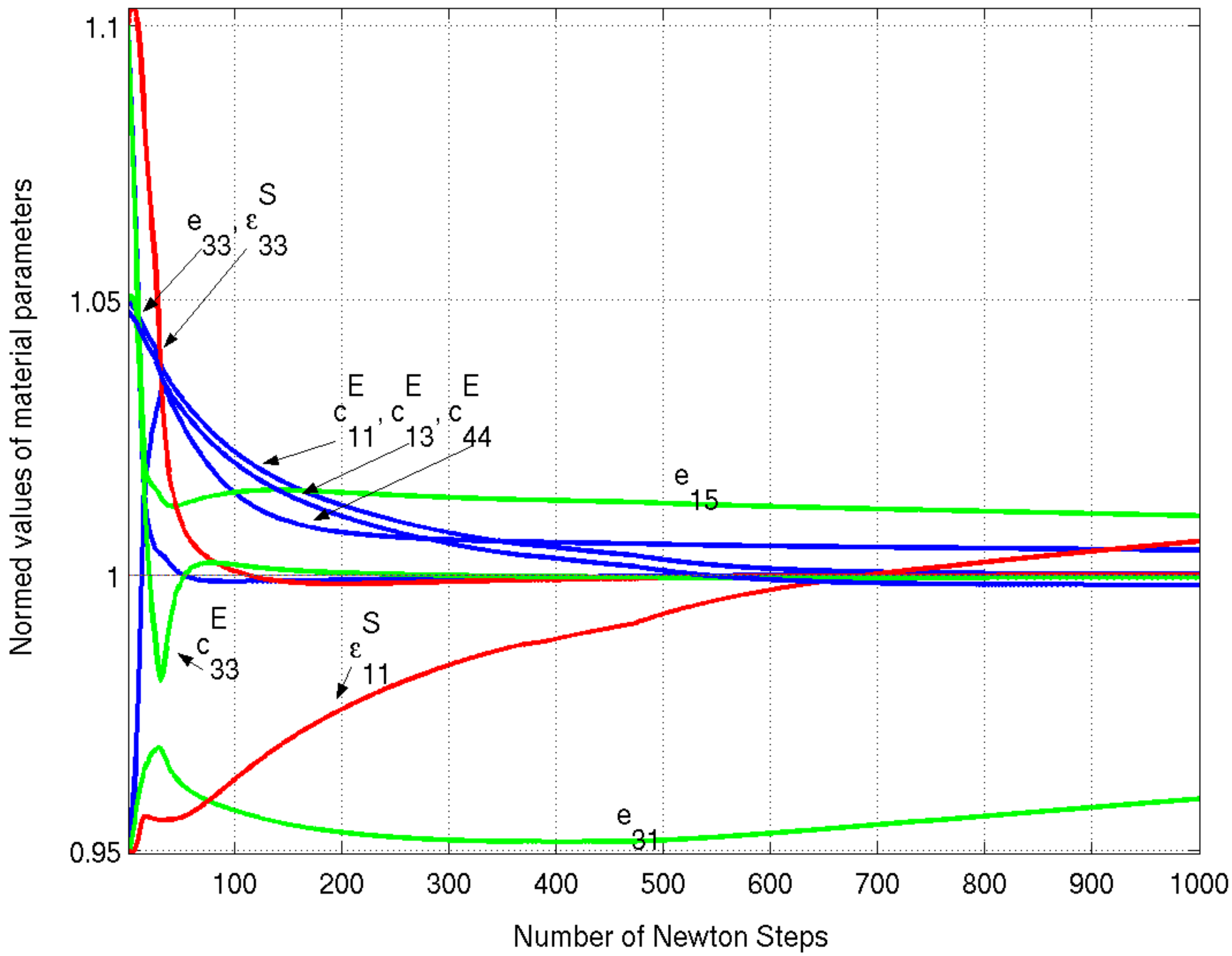
Numerical Results – Simultaneous Reconstruction of all parameters



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