Methods for Choosing the Regularization Parameter

An Overview of our Current Research

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Inverse Problems

- Many domains of modern technology require the solution of unstable problems
- In order to solve these problems one has to balance
  - Measurements which we cannot really trust
  - A-priori assumptions which we do not really know

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Regularization

- This balancing is done by regularization methods, e.g. Tikhonov:

$$x_{\mathrm{sol}} = \operatorname*{argmin}_x \begin{array}{c}\text{distance to}\\ \text{measurement}\end{array} + \alpha \begin{array}{c}\text{distance to}\\ \text{a-priori assump.}\end{array}$$

- One crucial point is finding the regularization parameter α

More Difficulties

- Noise structure badly known
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Selected Methods

- Cross-Validation
- L-Curve, Generalized Cross-Validation and others
- Morozov
- **Balancing principle**

Input

- Expectation of the noise with respect to the regularization parameter and measurement noise
- Regularized solutions with respect to the regularization parameters

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Where it works (provenly)

- Linear problems with almost all regularization methods
- Non-linear problems with some regularization methods
- Classical and Stochastic noise
- Metric solution spaces
- Some Multi-parameter regularization methods