Convergence Analysis and Error Reduction Properties of Adaptive Finite Element Methods for Distributed and Boundary Control Problems with Control Constraints

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Abstract

In this contribution, which is partially based on joint work with M. Hintermüller, we are concerned with the development, analysis and implementation of adaptive finite element methods for distributed and boundary optimal control problems with control constraints. The methods presented in this contribution provide an error reduction and thus guarantee convergence of the adaptive loop which consists of the essential steps 'SOLVE', 'ESTIMATE', 'MARK', and 'REFINE'. Here, 'SOLVE' stands for the efficient solution of the finite element discretized problems. The following step 'ESTIMATE' is devoted to a residual-type a posteriori error estimation of the global discretization errors in the state, the adjoint state, the control and the adjoint control. A bulk criterion is the core of the step 'MARK' to indicate selected edges and elements for refinement, whereas the final step 'REFINE' deals with the technical realization of the refinement process itself.
The analysis is carried out for a model problem using discretizations of the state and the adjoint state by continuous, piecewise linear finite elements and of the control and the adjoint control by elementwise constants with respect to a simplicial triangulation of the computational domain. Important tools in the convergence proof are the reliability of the estimator, a strong discrete local efficiency, and a quasi-orthogonality property. The proof does not require any regularity of the solution.
Numerical results illustrate the performance of the error estimator.