Multilevel preconditioning of graph Laplacians

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The goal of this talk is to present a multilevel preconditioning technology for symmetric discontinuous approximations of second order elliptic problems. Our approach is based on the following simple idea. The finite element space of piece-wise discontinuous polynomials of certain degree is projected onto the space of discontinuous piece-wise constant functions on the same partition. This constitutes the first (finest) space in the multilevel method.
The projection of the DG system on such a space is associated to the so-called "graph-Laplacian". In 2-D this is a very simple M-matrix with -1 on the n non-zero off diagonal entries, corresponding to the neighbouring FEs through the interfaces. We assume that the DG partition is obtained by a regular refinement of a given initial mesh generally consisting of both triangles and quadrilaterals.
We develop a unified concept of hierarchical splitting of the unknowns and using local analysis we derive uniform estimates for the related constant in the strengthen Cauchy-Bunyakowski-Schwarz (CBS) inequality.
This further is used to describe a multilevel preconditioner of the DG system. The preconditioner has uniformly bounded condition number and computational complexity proportional to the number of degrees of freedom.