Preconditioning of Discontinuous Galerkin (DG) finite element systems

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We present several DG methods for solving elliptic equations of second order. The goal of the presentation is to give an overview of the available recent results in preconditioning such systems.
We give short description of the results of Gopalakrishnan and Kanschat, 2003, Brenner and Zhao, 2005, Johannsen, 2005, and Lazarov, Vassilevski and Zikatanov, 2005. The results of the these works can be characterized as an application of the classical multigrid theory for construction and justification of optimal preconditioners. These results are restricted to existence of hierarchy of meshes and DG problems.
There is one novel twist in the last work, where the two-grid method is used to reduce the problem to a DG method on the space of discontinuous piecewise constant functions on the same triangulation. Such space does not have approximation properties and does not fit into the general theory of MG methods. The reduced problem has stiffness matrix called in graph theory "graph Laplacian". It has been shown that the two-grid method converges. Then the next important ingredient in the design of an optimal method is the construction of multilevel preconditioners for the "graph Laplacian". This is done in the framework of general theory of Algebraic Miltilevel Iteration methods of Axelsson and Vassilevski.