

Fast DD algorithms suitable for adaptive *hp* hierarchical and spectral discretizations of 3-*d* elliptic equations

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We focus on the Dirichlet-Dirichlet type *DD* methods for the discretizations by the finite elements, mapped from the square and cubic reference elements. The both types of basis polynomials are assumed, *i.e.*, defined by tensor products of the integrated Legendre's polynomials or by Lagrange interpolation polynomials over the "spectral" sets of nodes. For brevity, by spectral are implied the reference elements with the nodes of GLL (Gauss-Lobatto-Legendre) and GGC (Gauss-Lobatto-Chebyshev) quadratures. This is for the reason that as our algorithms, so their efficiency are similar for these Lagrange elements.

Main components of our *DD* algorithms implement optimal and suboptimal inexact solvers, including ones for local Dirichlet problems on subdomains of decomposition, local problems on their faces and the wire basket, and prolongation-restriction operations. All these components are based on the finite-difference/finite element (with low order elements) spectrally equivalent preconditioners for the reference element stiffness matrices. Orzag, 1980, introduced the preconditioner for spectral elements, which afterwards was theoretically justified by the efforts of Bernardi/Doudge/Maday, 1992, Canuto, 1994, and Casarin, 1997. For the hierarchical elements, the finite-difference preconditioners were derived by Korneev/Jensen, 1997. Fast solution procedures based on these preconditioners, *e.g.*, for local problems, appeared in 2000-2004. Among them only solvers for the hierarchical elements and one solver for the spectral elements have been theoretically approved (Korneev 2001,2002, Beuchler, 2002, Korneev/Xanthis/Anoufrieu, 2002, Beuchler/Schneider/Schwab, 2004). In 2005 Korneev/Rytov showed that the spectral element stiffness matrices have spectrally equivalent preconditioners, represented by the product of three matrices two of which are diagonal and the third possesses properties, allowing to adjust for it all fast solvers existing for the hierarchical elements. Modern shape of other components also resulted from several contributions, from which we mention papers of Widlund/Pavarino, 1996,1999, and Korneev/Langer/Xanthis, 2003.

We present algorithms which are suitable for adaptive computations. This means that for some minor restrictions, the powers of polynomials inside each finite element, each face and edge may be different. They may differ also in each reference direction of each element and each face. The resulting arithmetical cost of the *DD* solver, applicable to discretizations as by spectral so hierarchical elements, is $\mathcal{O}((\log N)^2(\log \log N)N)$ where N is the number on unknowns in the finite element system. The cost of multiplications by the global stiffness matrix is not taken into account in this estimate, although in a few cases it suits it.