Identification of Models for Nonlinearity and Hysteresis in Piezoelectricity

DFG Research Group (Emmy Noether Program)
Inverse Problems in Piezoelectricity

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Overview

- *Piezoelectric effect:*
  PDE model, material tensors

- *Determination of constant coefficients:*
  PDE based fit

- Identification of nonlinear material parameter curves:
  multiharmonic formulation

- Hysteresis identification
  iterative methods

Piezoelectric Transducers

Direct effect:	apply mechanical force	$\longrightarrow$ measure electric voltage
Indirect effect:	impress electric voltage	$\longrightarrow$ observe mechanical displacement

Application Areas:

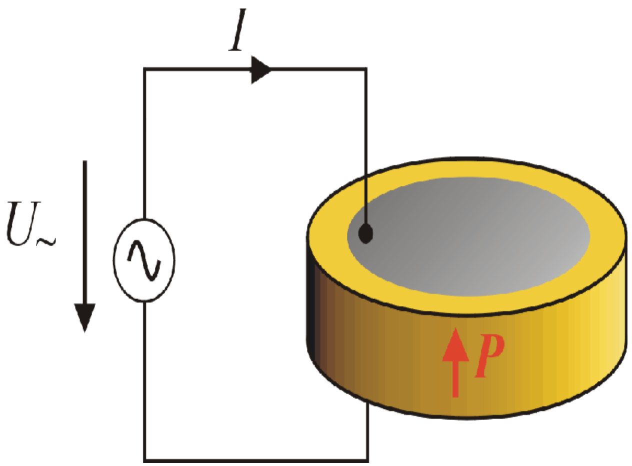
- Ultrasound (medical imaging & therapy)
- Force- and acceleration Sensors
- Actor injection valves (common–rail Diesel engines)
- SAW (surface-acoustic-wave) sensors
- . . .

Piezoelectric Effect

$$\begin{aligned} \vec{\sigma} &= \mathbf{c}^E \vec{S} - \mathbf{e}^T \vec{E} \\ \vec{D} &= \mathbf{e}\vec{S} + \varepsilon^S \vec{E} \end{aligned}$$

$$\begin{aligned}
\vec{\sigma} &= (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy})^T \ldots \text{stress} \\
\vec{S} &= (S_{xx}, S_{yy}, S_{zz}, S_{yz}, S_{xz}, S_{xy})^T = DIV^T \vec{d} \ldots \text{strain} \\
\vec{E} &= (E_x, E_y, E_z) = -\text{grad}\phi \ldots \text{electr. field} \\
\vec{D} &= (D_x, D_y, D_z)^T \ldots \text{electr. displacement} \\
\vec{d} &= (d_x, d_y, d_z) \ldots \text{mech. displacement} \\
\phi & \quad\ \ldots \text{electr. potential}
\end{aligned}$$

Piezoelectric PDEs:



$$\begin{aligned} \rho\frac{\partial^2\vec{d}}{\partial t^2} - DIV\left(\mathbf{c}^E DIV^T\vec{d} + \mathbf{e}^T \text{grad}\phi\right) &= 0 \quad \text{in } \Omega \\ -\text{div}\left(\mathbf{e} DIV^T\vec{d} - \varepsilon^S \text{grad}\phi\right) &= 0 \quad \text{in } \Omega \end{aligned}$$

Boundary conditions:

$$\begin{aligned} N^T\sigma &= 0 && \text{on } \partial\Omega \\ \phi &= 0 && \text{on } \Gamma_g \\ \phi &= \phi^e && \text{on } \Gamma_e \\ \vec{D}\cdot\vec{n} &= 0 && \text{on } \Gamma \end{aligned}$$

Γ_e . . . loaded electrode Γ_g . . . grounded electrode
$\Gamma = \partial\Omega \setminus (\Gamma_g \cup \Gamma_e)$ ϕ^e . . . impressend voltage

Fast forward solution $\longrightarrow$ Marcus Mohr [B.K. & Lahmer &Mohr, EJAM, to appear]

Simulation of piezoelectric transducers requires knowledge of material tensors $\mathbf{c}^E, \mathbf{e}, \varepsilon^S$

Material Tensors

$$
\left(\begin{array}{cccccc|ccc}
c_{11}^E & c_{12}^E & c_{13}^E & \cdot & \cdot & \cdot & \cdot & \cdot & e_{31} \\
c_{12}^E & c_{11}^E & c_{13}^E & \cdot & \cdot & \cdot & \cdot & \cdot & e_{31} \\
c_{13}^E & c_{13}^E & c_{33}^E & \cdot & \cdot & \cdot & \cdot & \cdot & e_{33} \\
\cdot & \cdot & \cdot & c_{44}^E & \cdot & \cdot & \cdot & e_{15} & \cdot \\
\cdot & \cdot & \cdot & \cdot & c_{44}^E & \cdot & e_{15} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2}(c_{11}^E - c_{12}^E) & \cdot & \cdot & \cdot \\
\hline
\cdot & \cdot & \cdot & \cdot & e_{15} & \cdot & \varepsilon_{11}^S & \cdot & \cdot \\
\cdot & \cdot & \cdot & e_{15} & \cdot & \cdot & \cdot & \varepsilon_{11}^S & \cdot \\
e_{31} & e_{31} & e_{33} & \cdot & \cdot & \cdot & \cdot & \cdot & \varepsilon_{33}^S
\end{array}\right)
$$

elasticity piezoelectric coupling dielectric

$\longrightarrow$ 10 different scalar coefficients

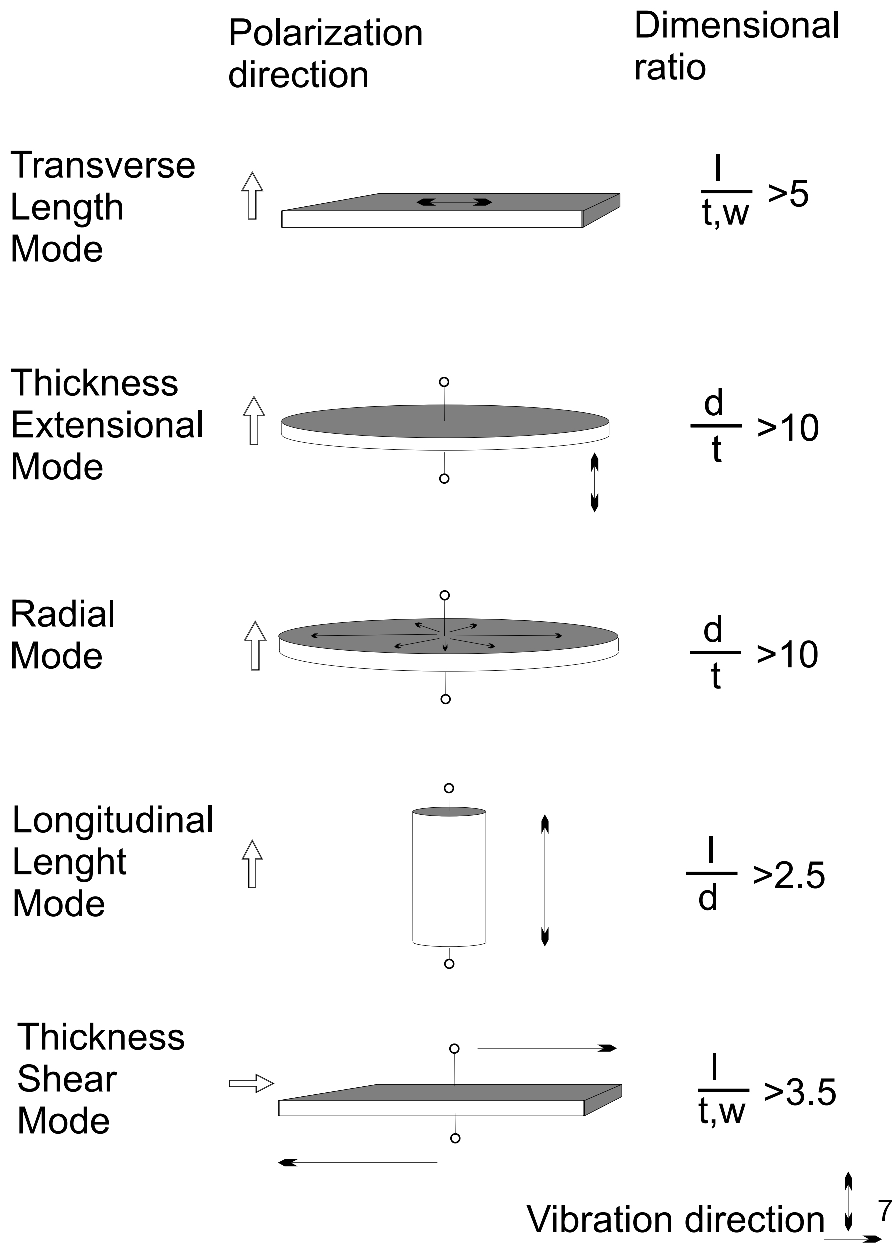
State of the Art:

1-d model simplification
via Test sample scheme

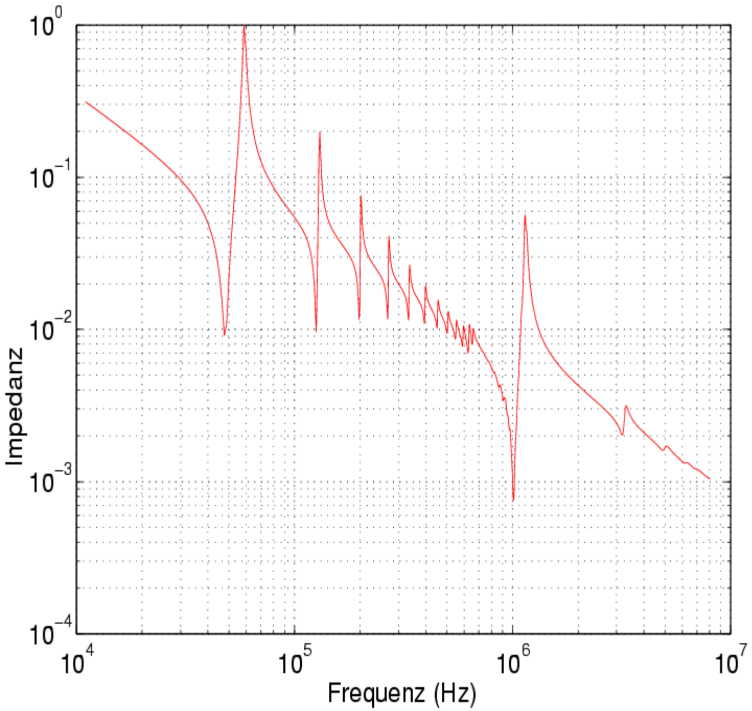
(IEEE Standard 1986, European Norm 1998)

→ explicit relation between material parameters and resonance frequencies

- costly: probes, measurements
- imprecise
- restricted to constant coefficients



Inverse Problem: Identification by Simulation of Piezo PDEs (I)



Find material tensors $\mathbf{c}^E$, $\mathbf{e}$, ε^S *from impedance measurements for different frequencies* ω

$$Z(\omega) = \frac{\hat{\phi}^e(\omega)}{j\omega\hat{q}(\omega)|_{\Gamma_e}}$$

$\hat{\phi}^e$. . . impressed voltage
$\hat{q}$ . . . surface charge

Inverse Problem: Identification by Simulation of Piezo PDEs (II)

$$\left.\begin{aligned} -\rho\omega^2\vec{\hat{d}} - DIV\left(\mathbf{c}^E DIV^T\vec{\hat{d}} + \mathbf{e}^T\text{grad}\hat{\phi}\right) &= 0 \quad \text{in } \Omega \\ -\text{div}\left(\mathbf{e}DIV^T\vec{\hat{d}} - \varepsilon^S\text{grad}\hat{\phi}\right) &= 0 \quad \text{in } \Omega \end{aligned}\right\} \quad \ldots \text{hom. harmonic PDE}$$

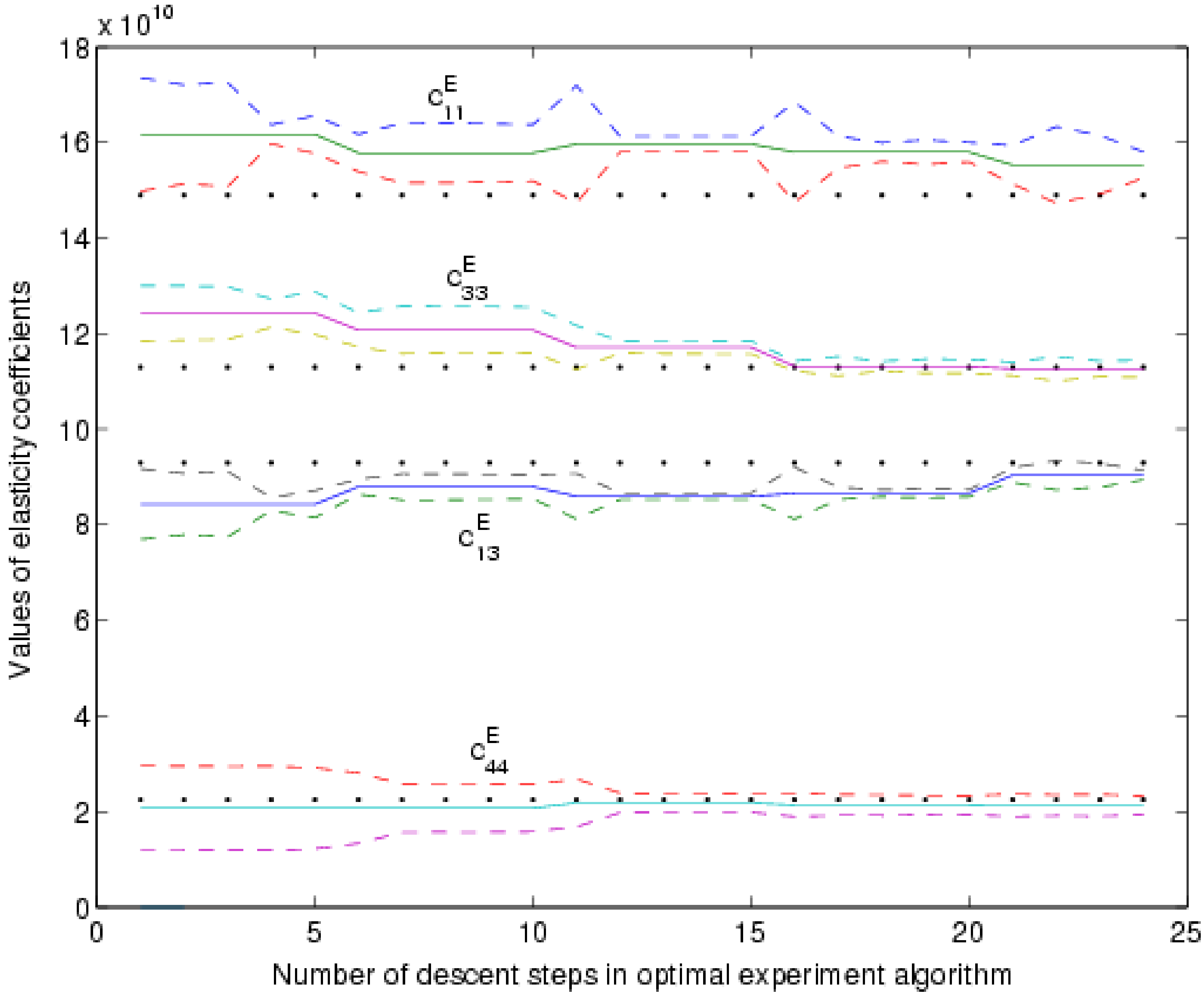
$$\hat{\phi} = \hat{\phi}^e \quad \ldots \text{inhom. BC}$$

$$\int_{\Gamma_e} \vec{D}\cdot\vec{n} = \hat{q} \quad \ldots \text{measurements}$$

- Nonlinear operator equation $F(\mathbf{c}^E, \mathbf{e}, \varepsilon^S) = \hat{q}$ where F contains PDE solution
- Solve by Newton's method: $F'(\mathbf{c}^E, \mathbf{e}, \varepsilon^S)[\underline{d\mathbf{c}^E}, \underline{d\mathbf{e}}, \underline{d\varepsilon^S}]$ via solution $(\underline{d\vec{\hat{d}}}, \underline{d\hat{\phi}})$ of

$$\begin{aligned} -\rho\omega^2\underline{d\vec{\hat{d}}} - DIV\left(\mathbf{c}^E DIV^T\underline{d\vec{\hat{d}}} + \mathbf{e}^T\text{grad}\underline{d\hat{\phi}}\right) &= -DIV\left(\underline{d\mathbf{c}^E} DIV^T\vec{\hat{d}} + \underline{d\mathbf{e}^T}\text{grad}\hat{\phi}\right) \\ -\text{div}\left(\mathbf{e}DIV^T\underline{d\vec{\hat{d}}} - \varepsilon^S\text{grad}\underline{d\hat{\phi}}\right) &= \text{div}\left(\underline{d\mathbf{e}}DIV^T\vec{\hat{d}} - \underline{d\varepsilon^S}\text{grad}\hat{\phi}\right) \\ \underline{d\hat{\phi}} &= 0 \quad \text{on } \Gamma_e \end{aligned}$$

- optimum experiment design: choice of measurement frequencies
- sensitivity matrix $F'(\mathbf{c}^E, \mathbf{e}, \varepsilon^S)$ yields confidence intervals


x 10^10
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
c^E_11
c^E_33
c^E_13
c^E_44
Values of elasticity coefficients
Number of descent steps in optimal experiment algorithm

Nonlinear dependence

Large excitations (actuator applications):

$$\begin{array}{rcl} \rho\frac{\partial^2\vec{d}}{\partial t^2} - DIV\left(\mathbf{c}^E(S)DIV^T\vec{d} + \mathbf{e}(S,E)^T\text{grad}\phi\right) & = & 0 \\ -\text{div}\left(\mathbf{e}(S,E)DIV^T\vec{d} - \varepsilon^S(E)\text{grad}\phi\right) & = & 0 \end{array}$$

$$S = |DIV^T\vec{d}| \qquad E = |\text{grad}\phi|$$

$\rightarrow$ infinite dimensional problem, instability

Multiharmonic Formulation (I)

Linear case:

$$\begin{aligned} -\rho\omega^2\hat{\vec{d}} - DIV\left(\mathbf{c}^E DIV^T\hat{\vec{d}} + \mathbf{e}^T \text{grad}\hat{\phi}\right) &= 0 \\ -\text{div}\left(\mathbf{e}DIV^T\hat{\vec{d}} - \varepsilon^S \text{grad}\hat{\phi}\right) &= 0 \end{aligned}$$

Excitation at frequency ω

$\longrightarrow$ spectrum of $\vec{d}$, ϕ concentrated to ω.

Nonlinear case:

$\mathbf{c}^E(|DIV^T\vec{d}|)$, $\mathbf{e}(|DIV^T\vec{d}|, |\text{grad}\phi|)$, $\varepsilon^S(|\text{grad}\phi|)$

Higher harmonics appear

$\longrightarrow$ multiharmonic Ansatz:

$$\vec{d}(\vec{x},t) \approx \sum_{n=-N}^{N} e^{jn\omega t}\hat{\vec{d}}_n(\vec{x}) \qquad \phi(\vec{x},t) \approx \sum_{n=-N}^{N} e^{jn\omega t}\hat{\phi}_n(\vec{x})$$

[Bachinger, Schöberl, Langer], nonlinear magnetics

Multiharmonic Formulation(II)

Model problem

$$d_{tt} - (c(d_x)\ d_x)_x = f$$

Multiharmonic Ansatz

$$d(x,t) \approx \sum_{n=-N}^{N} e^{jn\omega t} \hat{d}_n(x)$$

Insert into nonlinear PDE and test with $\frac{\omega}{2\pi} e^{-jk\omega t}$

Orthogonality

$$\frac{\omega}{2\pi} \int_0^{2\pi/\omega} e^{jn\omega t} e^{-jk\omega t}\, dt = \begin{cases} 1 & \text{if } k = n \\ 0 & \text{else} \end{cases}$$

$$-\omega^2 k^2 \hat{d}_k(x) - \left(\frac{\omega}{2\pi} \sum_{n=-N}^{N} \int_0^{2\pi/\omega} c(d_x(x,t)) e^{jn\omega t} e^{-jk\omega t}\, dt\ \hat{d}_{n\,x}(x) \right)_x = 0$$

Multiharmonic formulation (III)

$$-\omega^2 k^2 \hat{d}_k(x) - \left(\frac{\omega}{2\pi} \sum_{n=-N}^{N} \int_0^{2\pi/\omega} c(d_x(x,t)) e^{jn\omega t} e^{-jk\omega t}\, dt\, \hat{d}_{n\,x}(x) \right)_x = 0$$

Polynomial Approx. $c(d_x(x,t)) \approx \sum_{p=0}^{P} a_p \left(\sum_{n=-N}^{N} e^{jn\omega t} \hat{d}_{x\,n}(x) \right)^p$

Multinomial Thrm. $\left(\sum_{n=-N}^{N} e^{jn\omega t} \hat{s}_n \right)^p = \sum_{\mathbf{p}=(p_0,\ldots,p_N)} \binom{p}{\mathbf{p}} e^{j(\sum n p_n)\omega t} \cdot \hat{s}_{-N}^{p_{-N}} \cdots \hat{s}_N^{p_N}$

$$\int_0^{2\pi/\omega} c(d_x(x,t)) e^{jn\omega t} e^{-jk\omega t}\, dt = \sum_{p=0}^{P} a_p \bar{c}^p_{n-k}(\hat{d}_{0\,x}(x), \ldots, \hat{d}_{N\,x}(x))$$

by factoring out $e^{jm\omega t}$ and using orthogonality.

Identification in Multiharmonic Formulation

Determine $c(\lambda) \approx \sum_{p=0}^{P} a_p \lambda^p$ *from boundary measurements of* $\hat{d}_{-N}, \ldots, \hat{d}_N$ *solving the PDE system*

$$-\omega^2 k^2 \hat{d}_k - \left(\sum_{n=-N}^{N} \sum_{p=0}^{P} a_p \bar{c}_{n-k}^{p}(\hat{d}_0, \ldots, \hat{d}_N) \hat{d}_{n\,x} \right)_x = 0\,.$$

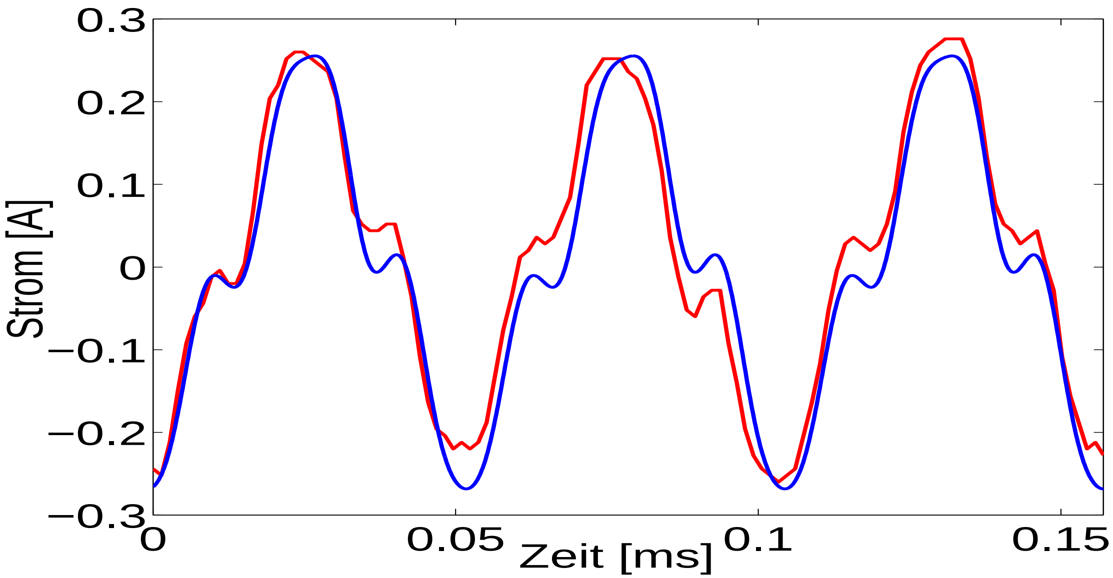
Compute polynomial coefficients $a_0, \ldots, a_P$ of c by fitting measurements and simulations of $\hat{d}_{-N}, \ldots, \hat{d}_N$ using Newton's method

- Arbitrary good approximation of c for $N, P \to \infty$;
- Regularization by finite dimensionality $N, P < \infty$;
- Multilevel strategy applicable: sucessively solve problems with increasing N, P
- Applicable to 3D complex valued piezoelectric PDEs.

Multiharmonic Formulation: Data smoothing

— . . . measurement

— . . . approximation with $N = 6$ higher harmonics



→ **Motivation for regularization by discretization**

[Natterer 77], [Vainikko, Hämarik 85], [Engl,Neubauer 87], [Groetsch,Neubauer 88], [Kirsch 96], [Pereverzev 00], nonlinear case: [BK 00-]

Numerical Test Example

1D piezoelctric PDEs

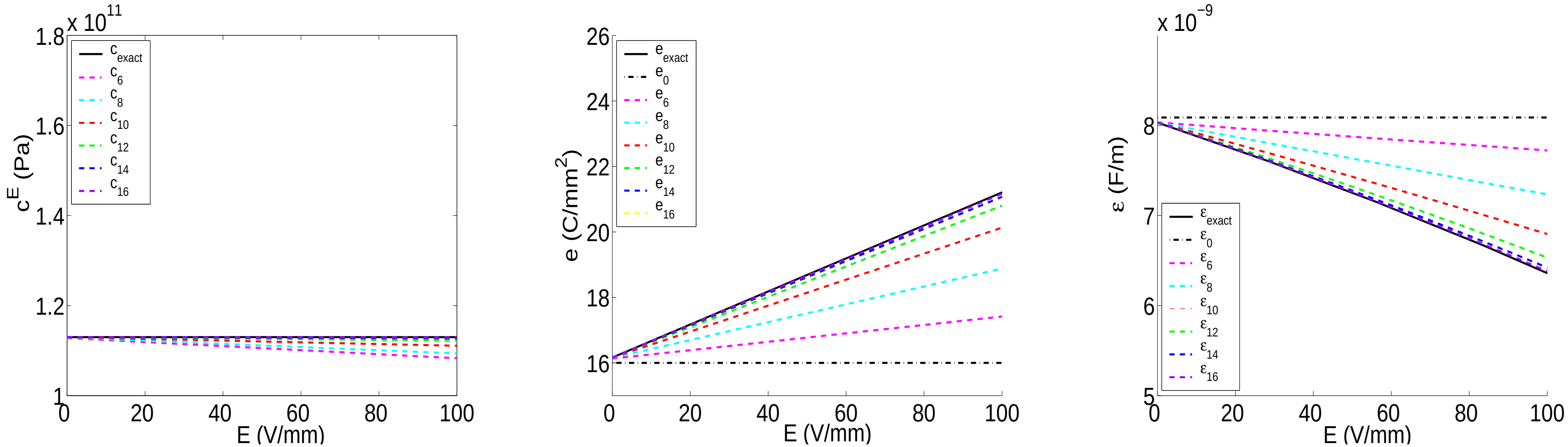
$$\begin{aligned} \rho d_{tt} - \left(\mathbf{c}^E d_x + \mathbf{e}(|\phi_x|)^T \phi_x\right)_x &= 0 \\ -\left(\mathbf{e}(|\phi_x|) d_x - \varepsilon^S(|\phi_x|)\phi_x\right)_x &= 0 \end{aligned}$$

- Pz27 material (Ferroperm Piezoceramics A/S);
- Synthetic data from [Andersen et al., 2000] on large signal behaviour of Pz27
- Starting values: material constants for small-signal behaviour from Ferroperm datasheet

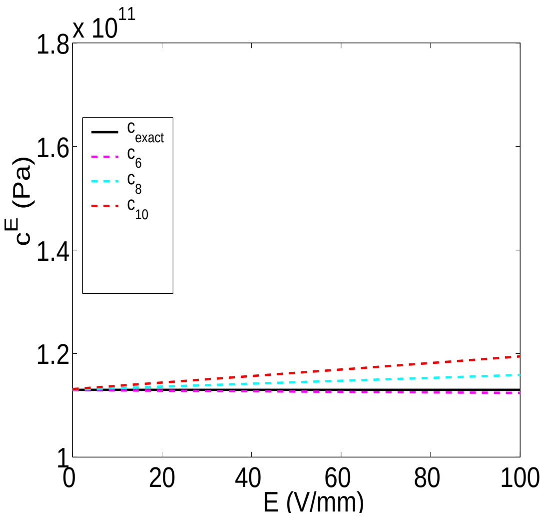
Numerical Results

Simultaneous identification of c_{33}^E, e_{33} and ε_{33}^S by Newton's method, applied to the multiharmonic formulation of the piezoelectric PDEs

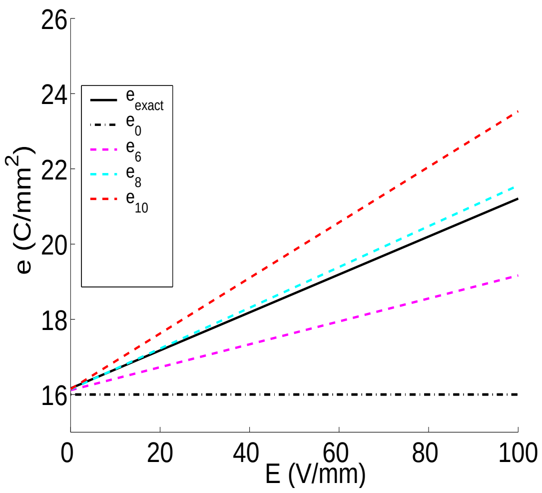
with exact data:



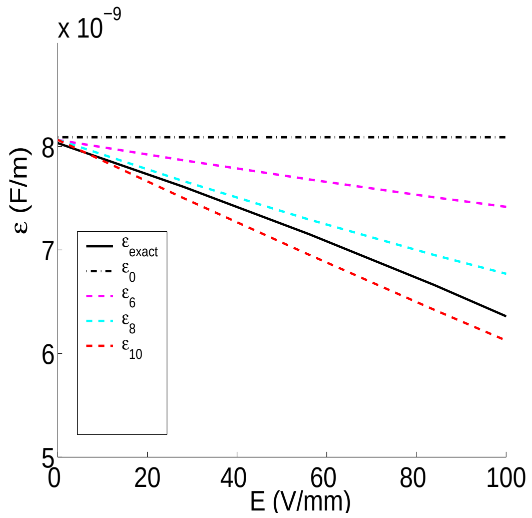
with 1% noise:



x 10^11
1.8
1.6
1.4
1.2
1
c^E (Pa)
c_{exact}
c_6
c_8
c_{10}
0 20 40 60 80 100
E (V/mm)

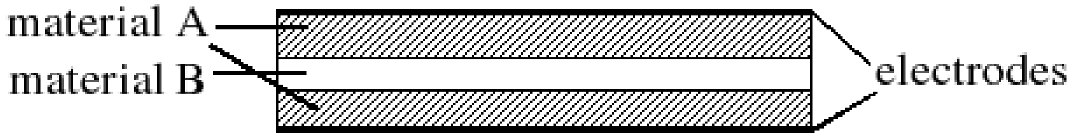


26
24
22
20
18
16
e (C/mm^2)
e_{exact}
e_0
e_6
e_8
e_{10}
0 20 40 60 80 100
E (V/mm)



x 10^-9
8
7
6
5
ε (F/m)
ε_{exact}
ε_0
ε_6
ε_8
ε_{10}
0 20 40 60 80 100
E (V/mm)

Tests for Layered Media



PDEs:

$$\begin{aligned} \rho d_{tt} - (c_A(d_x)d_x)_x &= 0 \qquad x \in [0, L_1] \cup [L_1 + L_2, L_1 + L_2 + L_3],\ t \in [0, T] \\ \rho d_{tt} - (c_B(d_x)d_x)_x &= 0 \qquad x \in [L_1, L_1 + L_2],\ t \in [0, T] \end{aligned}$$

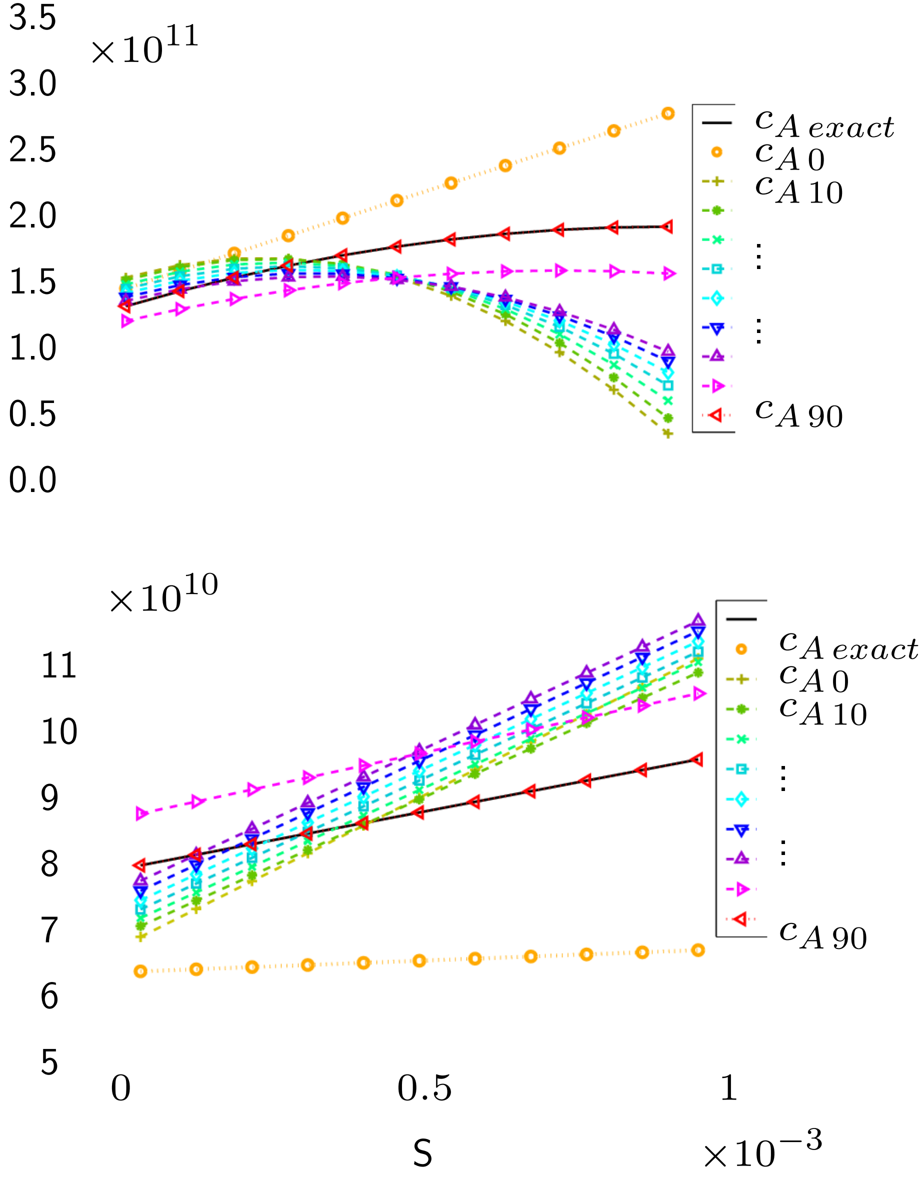
boundary conditions:

$$(c_A(d_x)d_x)(0, t) = g(t)\ , \qquad (c_A(d_x)d_x)(L_1 + L_2 + L_3, t) = g(t)\,,$$

measurements:

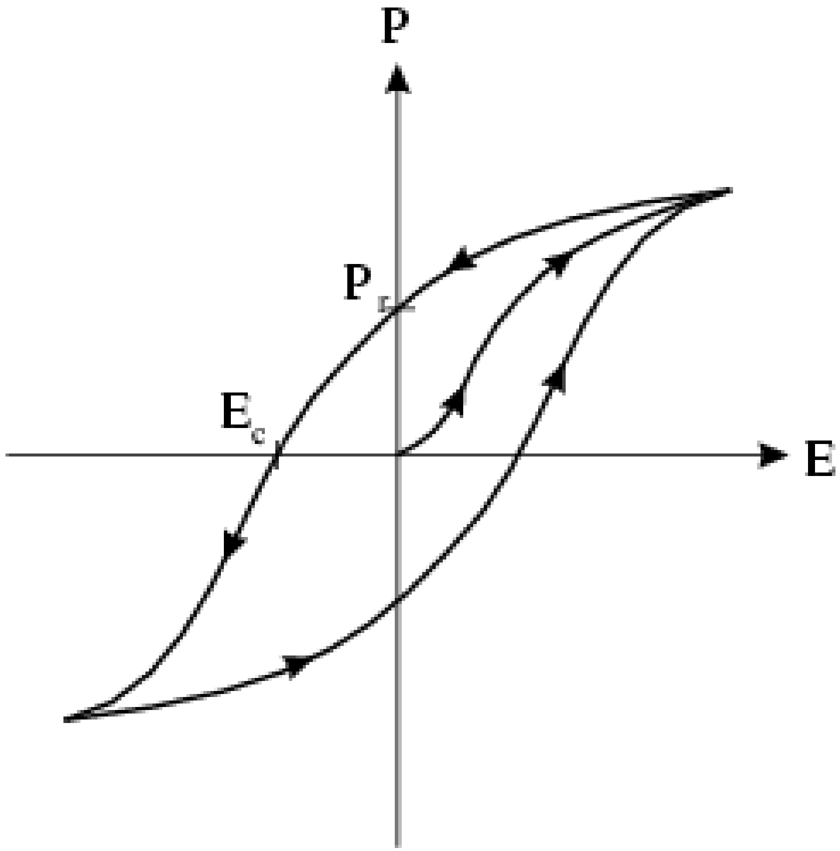
$$d(L, t) = f(t)\,.$$

Convergence, simultaneously for c_A (top) and c_B (bottom)



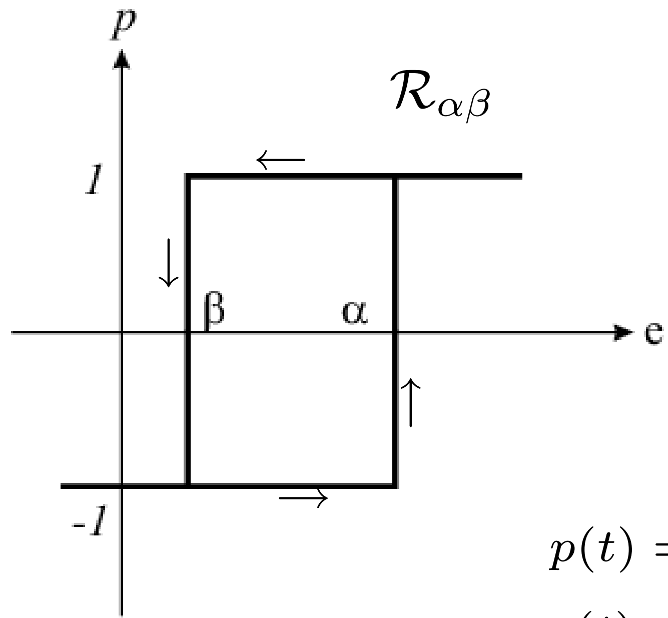
Hysteresis Identification

$$\vec{D} = \mathbf{e}\vec{S} + \varepsilon_0^S \vec{E} + \vec{P}$$



Preisach Model:

$$p(t) = \iint_{-1\leq\beta\leq\alpha\leq 1} w(\alpha,\beta)\mathcal{R}_{\alpha,\beta}[e](t)\, d\alpha d\beta$$



$$p(t) = \frac{P(t)}{P_{\max}}$$

$$e(t) = \frac{E(t)}{E_{\max}}$$

Rate independence,

Volterra property,

Congruency of hysteresis loops,

Wipe out (memory deletion),

$\Rightarrow$ Shape (Everett) function s exists such that

$$p(t) = \iint_{-1\leq\beta\leq\alpha\leq 1} w(\alpha,\beta)\mathcal{R}_{\alpha,\beta}[e](t)\,d\alpha d\beta = \frac{1}{2}s(-e_0,e_0) + \sum_{k=1}^{N} s(e_i,e_{i+1})$$

where $e_0, e_1 \ldots e_N$ are the preceding "dominant" extremal input values, $e_{N+1} := e(t)$

Forward problem:

Input-output model: In each time step, evaluate Preisach operator in all space points:

- *apply deletion rules*
- *compute Everett sum*

Hysteresis in PDEs: involves implicit iteration of hysteretic input-output model

Inverse Problem:

Determine weight function w or shape function s in Preisach operator $\mathcal{P}$.

Iterative Hysteresis Identification

Model problem:

$$
(*) \begin{cases} \text{PDE :} & \rho d_{tt} - (\mathcal{P}[d_x])_x = 0 \quad x \in [0, L],\ t \in [0, T] \\ \text{boundary conditions:} & d(0,t) = f_0(t) \qquad d(L,t) = f_L(t) \\ \text{initial conditions:} & d(x,0) = d_0(x) \qquad d_t(x,0) = d_1(x) \end{cases}
$$

$$
(**) \text{measurements:} \quad (\mathcal{P}[d_x])\,(L,t) = g(t)
$$

a) Identify by alternating iterations: Each step consists of

- solve measurements (**) for $\mathcal{P}$.
- solve PDE (*) for d

b) Apply Newton's method to (**) as an equation $F(\mathcal{P}) = g$ with F containing solution of (*).

Alternating iteration:

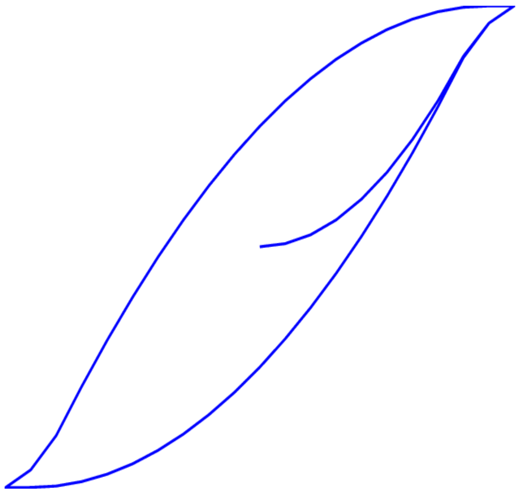
n=0: choose starting values for d and $\mathcal{P}$
for $n = 0, 1, 2, 3, \ldots$
fit measurements $(\mathcal{P}[d_x^{(n)}])(L,t) = g(t) \to \mathcal{P}^{n+1}$
solve PDE with hysteresis $\mathcal{P}^{n+1} \to d^{n+1}$

Newton iteration for $F(\mathcal{P}) = g$:

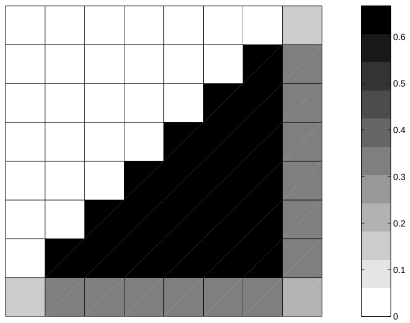
n=0: choose starting values for $\mathcal{P}$
for $n = 0, 1, 2, 3, \ldots$
evaluate F:
solve PDE with hysteresis $\mathcal{P}^n$
compute Newton step $\Delta\mathcal{P}^n = F'(\mathcal{P}^n)^{-1}(g - F(\mathcal{P}^n))$:
solve linearized PDE and fit measurements (iterative coupling):
$\mathcal{P}^{n+1} = \mathcal{P}^n + \Delta\mathcal{P}^n$.

Test Example: Hysteresis with Saturation

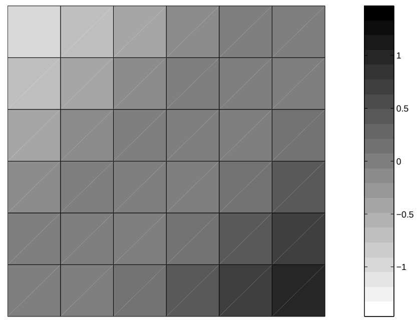
Main hystersis loop:



weight function w:

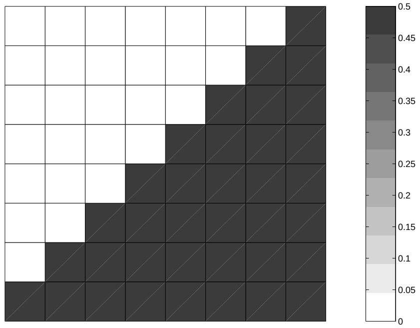


shape function s:

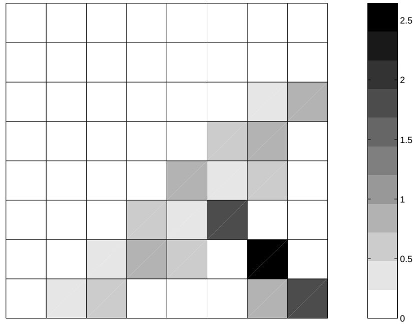


Numerical Results: Hysteresis with Saturation, Weight fcn.

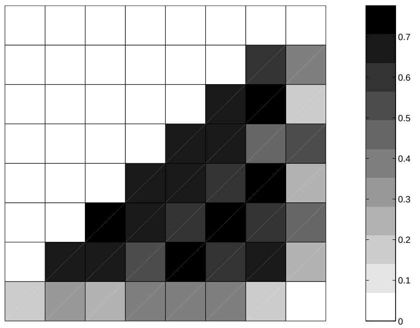
w_0:



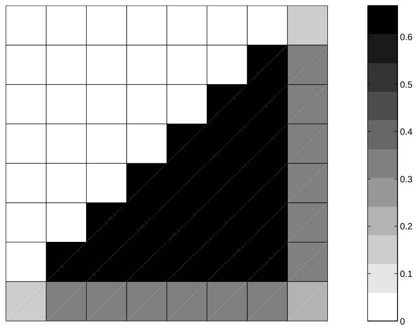
w_1:



w_2:

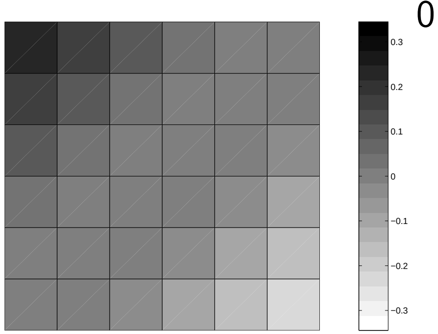


w_3:



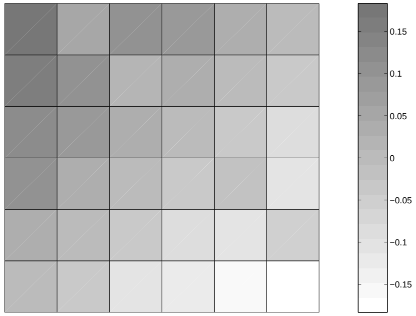
Numerical Results: Hysteresis with Saturation, Everett fcn.

$s_0 - s$:



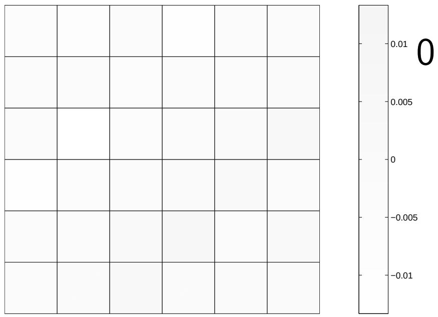
0.3

$s_1 - s$:



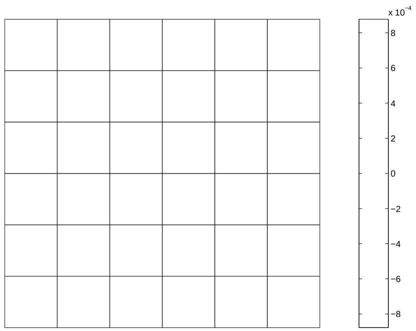
0.15

$s_2 - s$:



0.01

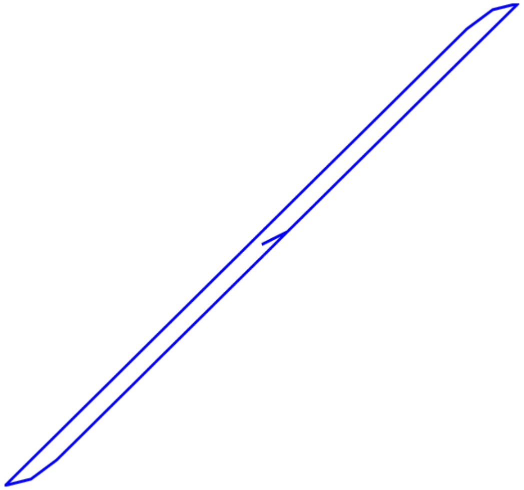
$s_3 - s$:



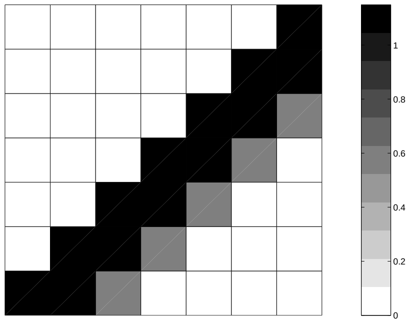
1.e-4

Test example: Narrow Hysteresis

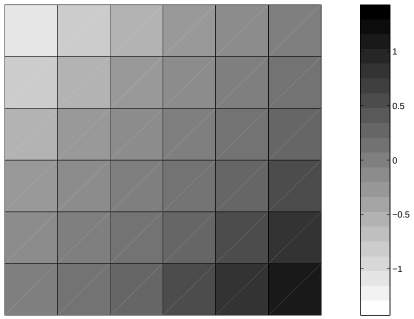
Main hystersis loop:



weight function w:

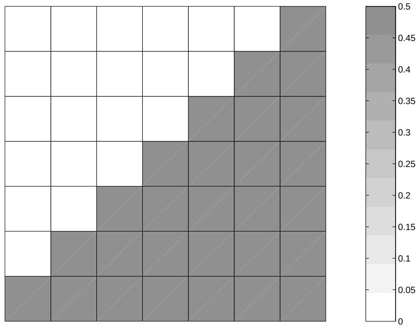


shape function s:

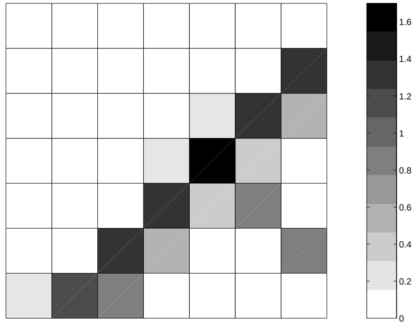


Numerical Results: Narrow Hysteresis

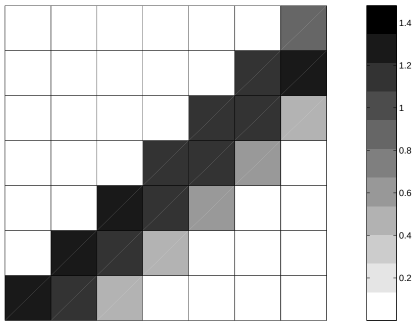
w_0:



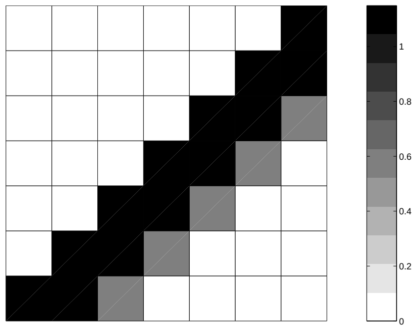
w_1:



w_2:

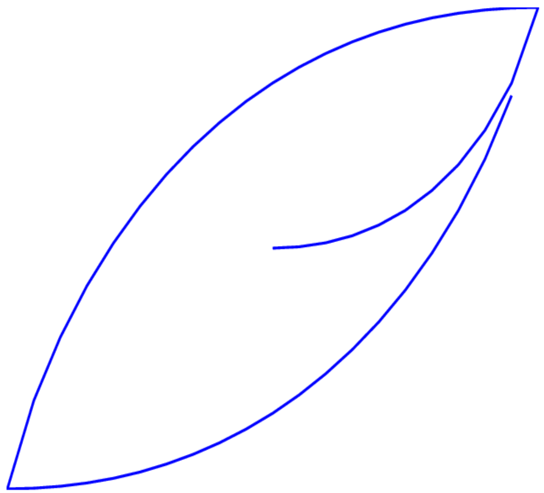


w_3:

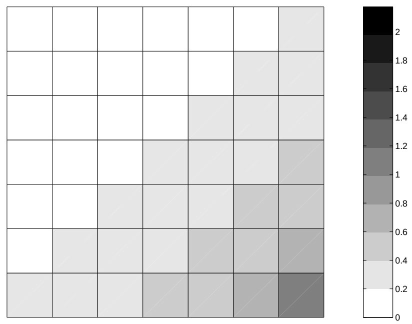


Test example: Broad Hysteresis

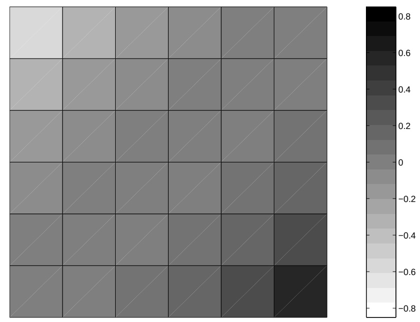
Main hystersis loop:



weight function w:

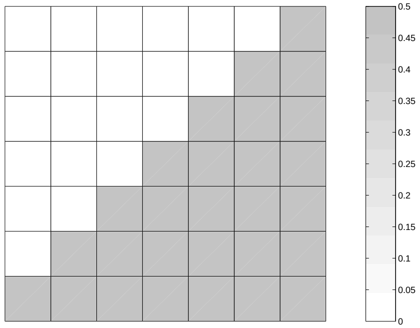


shape function s:

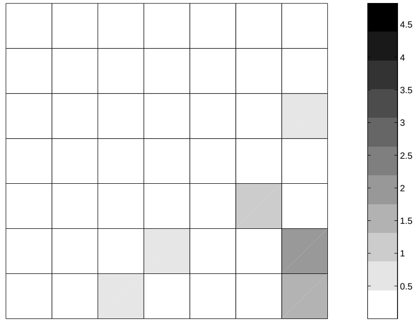


Numerical Results: Broad Hysteresis

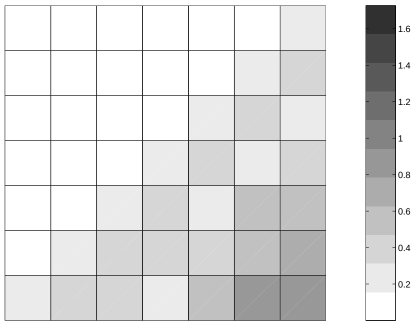
w_0:



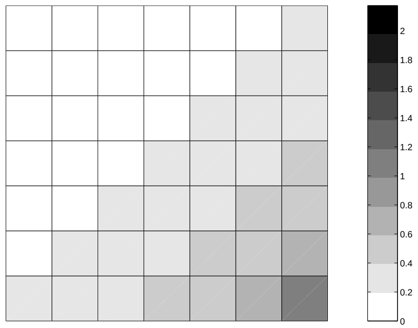
w_1:



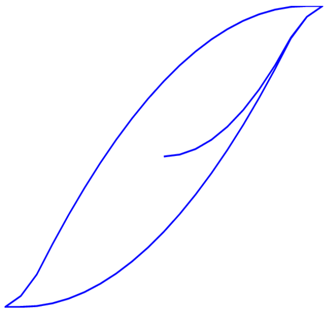
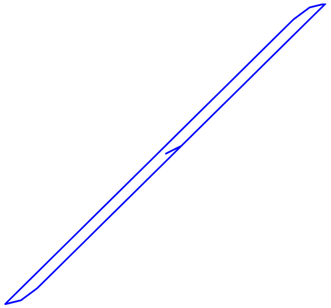
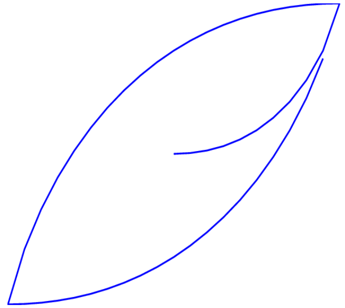
w_2:



w_3:



Comparison of Alternating Iteration versus Newton

				
alternating iteration	5 39.63 1.6e-4	5 39.29 1.8e-4	5 40.73 0.9e-4	no. steps CPU sec. residual
Newton	3 (1.3) 35.09 2.8e-4	4 (1.5) 49.58 0.9e-4	4 (1.25) 45.63 1.9e-4	no. steps (av. inner its.) CPU sec. residual

Conclusions and Outlook

- PDE based identification of material parameters
- Nonlinearity
- Hysteresis

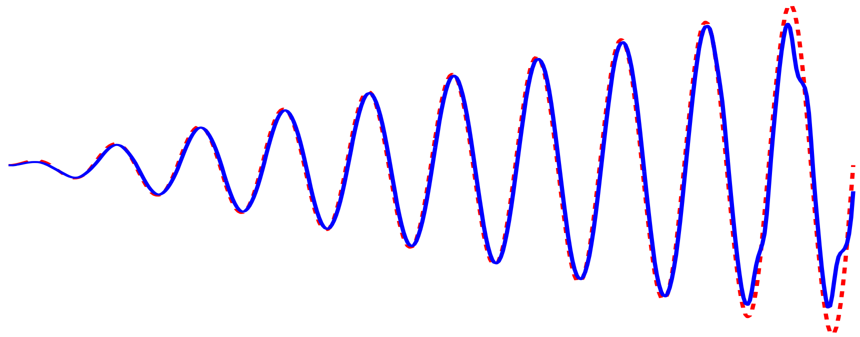
$\hookrightarrow$ 3D implementation of multiharmonic parameter id.

$\hookrightarrow$ Hysteresis modelling in piezoelectricity

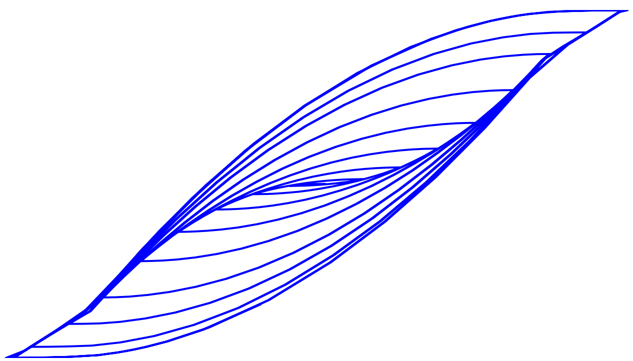
$\longrightarrow$ Temperature dependence

Full PDE Model versus Input-Output Simplification (I)

Estimated $d_x^{est}(L,t) := \frac{d(0,t)-d(L,t)}{L} = \frac{f_L(t)-f_0(t)}{L}$ - -
versus correct values $d_x(L,t)$ –:



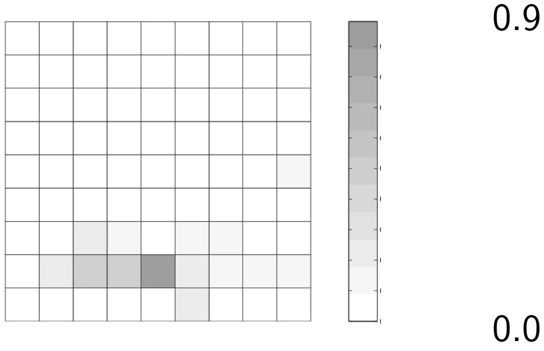
Hysteresis curve $\mathcal{P}[d_x](L,t)$:

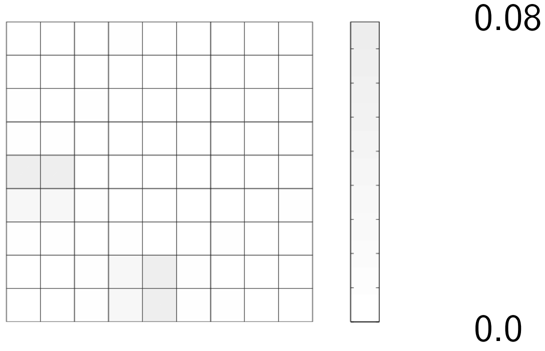


Full PDE Model versus Input-Output Simplification (II)

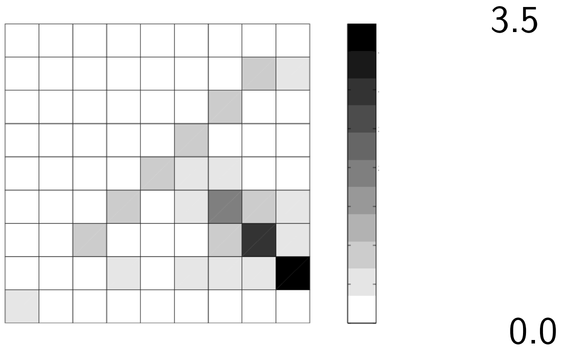
Difference between exact and identified w (left) and s (right)

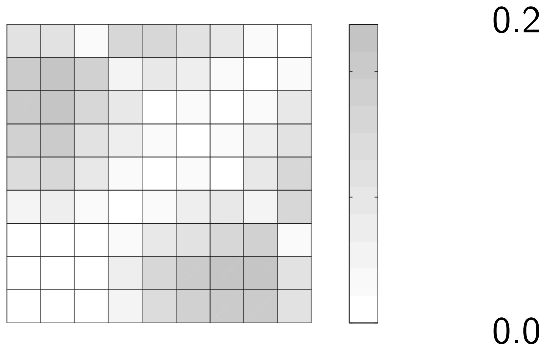
a) in PDE model:





b) with simplification $u_x(L,t) \approx u_x^{est}(L,t)$:





$\rightarrow$ Depending on the ratio $\rho/c\left(\frac{L}{T}\right)^2$, it can be necessary to identify within the PDE.

$\rightarrow$ Shape function identification is less ill-posed than weight function identification