Application of Moreau-Yoshida Theorem to Elastoplasticity

Peter Gruber, Johanna Kienesberger, Jan Valdman

Special Research Program SFB F013
'Numerical and Symbolic Scientific Computing',
Johannes Kepler University Linz,
Altenbergerstrasse 69, A-4040 Linz, Austria

e-mail: `Jan.Valdman@sfb013.uni-linz.ac.at`

The main task of our project is concerned with the development of multilevel solvers for elastoplasticity. The governing equations describing elastoplasticity are then the equilibrium of forces, the non-linear relationship between strain and stress, the linear dependence of the strain on the displacements and the Prandtl-Reuß normality law describing the time evaluation of plastic materials. By discretizing the time derivatives occurring in the normality law and calculating the weak dual formulation, the problem can be considered as a variational inequality in each time step. An equivalent formulation to the variational inequality is the optimization problem in the displacement u and the plastic part of the strain p

$$f(u,p) = \min_{v,q} f(v,q)$$

under incompressibility constraints. The functional f depends on the displacement u smoothly and on the plastic part of the strain p non-smoothly. It reads

$$f(u,p) = \frac{1}{2}||\varepsilon(u) - p||_C^2 + \psi(p) + \text{linear terms in } u$$

with $\psi(p)$ convex and non-smooth, $||x||_C^2 := (Cx){:}x$ and $:$ being the scalar product of matrices and the well known elasticity tensor C. In analytical calculation a function $\hat{p}(u)$ can be found explicitly, which yields $f(u, \hat{p}(u)) = \min_p f(u,p)\ \forall u$. The Moreau-Yoshida theorem from convex analysis states that the functional $\hat{f}(u) := \min_p f(u,p) = f(u, \hat{p}(u))$ is Fréchet-differentiable in u with $\hat{f}'(u) = \langle \varepsilon(u) - \hat{p}(u), \cdot \rangle_C$. Thus applying Moreau-Yoshida theorem leads to a smooth minimization problem in the displacement u only.

In this talk we concentrate on the explanation of the Moreau-Yoshida technique in elastoplasticity and report on some numerical experiments.