Numerical Instabilities of Structural Finite Elements at Large Strains

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Efficiency of standard displacement finite elements is often severely limited due to locking phenomena. For two-dimensional and three-dimensional solids these are in particular (in-plane) shear locking and volumetric locking. It is therefore customary to avoid these locking effects by applying reduced integration or mixed methods. These concepts allow efficient application of low-order displacement based elements also for thin-walled structures or nearly incompressible material behavior. Among these concepts, the enhanced assumed strain (EAS) method enjoys particular popularity.

It has been found that, unfortunately, EAS elements tend to exhibit spurious instabilities ("hourglassing") in the presence of large compressive strains, see for instance Wriggers and Reese [1]. Since then, a couple of remedies and alternative formulations have been proposed (de Souza Neto et al. [2], Wall et al. [3], Reese and Wriggers [4], among others), aspiring to remove artificial instabilities while retaining locking-free behavior.

A fundamental difficulty in this discussion is the circumstance that, to the authors' best knowledge, there is neither a proper definition nor – consequently – a mathematical analysis of stability of finite elements for large deformations. As a result, methods to avoid *numerical* instabilities may tend to ignore *physical* instabilities as well. The present contribution therefore highlights the mechanical origin of the artificial instabilities.

Inspired by this phenomenological analysis, a new stabilized finite element formulation for large strain analysis is presented. It turns out that some sort of compromise between element flexibility and stability has to be found. The crucial question concerns appropriate criteria for a desired element behavior and their realization within the framework of a stabilized method. In numerical experiments, using hyperelastic material models, the presented approach is compared to higher order finite element formulations. Results suggest that the proposed method effectively removes hourglassing without neglecting physical instabilities.

References

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