# An important interrelation between fast solvers for spectral and hierarchical $hp$ discretizations

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The stiffness matrices for the two types cubic $p$ reference elements, called *spectral* and *hierarchical*, are compared. The coordinate functions of the spectral elements are the Lagrange interpolation polynomials induced by the nodes of Gauss-Lobatto-Legendre or Gauss-Lobatto-Chebyshev quadrature rules. The tensor products of the integrated Legendre's polynomials are assumed for the coordinate functions of the hierarchical reference elements. It is well known that the computational costs of solvers for these stiffness matrices are of crucial importance for the efficiency of the DD (domain decomposition) algorithms for $hp$ discretizations. We show that, by a simple transform, the stiffness matrix of one type reference element may be transformed into the matrix with the properties, similar to the properties of the other type reference element stiffness matrices. This allows, *e.g.,* to adapt all fast solvers, designed for the hierarchical reference elements, to the spectral reference elements stiffness matrices and vice versa without lost of the efficiency. For instance, in this way we adapt the fast DD type solver of Korneev (2002) and fast multiresolution wavelet solver of Beuchler/Schneuder/Schwab (2004), suggested for the hierarchical reference elements, to the stiffness matrices of spectral reference elements. Combining these solvers with some similarly designed fast solvers for faces, we come to the fast DD solvers for the spectral discretizations of the 2-nd order elliptic equations in 3-$d$ domains. In the case of the shape quasiuniform discretization and element wise coefficients, the total number of arithmetic operations required for obtaining the finite element solution with the accuracy $\epsilon$ in the energy norm is $\mathcal{N}_{DD} \leq c\,(1 + \log p)^2 \mathcal{R} p^3 \log \epsilon^{-1}$, where $\mathcal{R}$ is the number of finite elements. The cost of the multiplications in the iteration process by the stiffness matrix of the finite element discretization is not taken into account. Generalizations of some results of the multiresolution wavelet analysis on the nonuniform meshes are other consequences.