Scalable FETI Based Algorithms for Numerical Solution of Contact Problems

Z. Dostál and D. Horák
FEI VŠB-Technical University of Ostrava, Tř. 17. listopadu,
CZ-70833 Ostrava, Czech Republic
zdenek.dostal@vsb.cz
D. Stefanica
Baruch College, City University of New York, NY 10010, U.S.A.

We shall first briefly review the FETI based domain decomposition methodology adapted to the solution of variational inequalities such as those that describe equilibrium of a system of bodies in mutual contact. We shall consider the classical FETI with natural coarse grid, its new variant Total FETI and FETI-DP. As a result, we shall obtain a convex quadratic programming problem with bounds and equality constraints. Adapting the classical results concerning the solution of linear elliptic boundary value problems, we shall show that the condition number of the Hessian matrices of the related quadratic forms are independent on the discretization parameter h.

Then we shall present two algorithms for the solution of resulting quadratic programming problems. The unique feature of these algorithms is their capability to solve the class of quadratic programming problems with spectrum in a given positive interval in O(1) iterations. The algorithms enjoy the rate of convergence that is independent of conditioning of constraints and the results are valid even for linearly dependent equality constraints.

Finally we put together our results on approximation of variational inequalities and those on quadratic programming to develop algorithms for the solution of both coercive and semi-coercive variational inequalities. We shall show that the algorithms are scalable. Rather surprisingly, the results are qualitatively the same as the classical results on scalability of FETI for linear elliptic problems. We give results of numerical experiments with parallel solution of both coercive and semicoercive problems discretized by up to more than eight million of nodal variables to demonstrate numerically scalability of the algorithms presented that was predicted by the theory.

References

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