RICAM Special Semester on Optimization

Workshop 6
Conic and Copositive Optimization

Book of Abstracts


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Linz, Austria
9–11 December 2019

Talks Monday 9 December 2019

9:00	Welcome and Opening
	Tamás Terlaky: *Parametric Analysis of Semidefinite and Second Order Conic Optimization*
	Gábor Pataki: *On positive duality gaps in semidefinite programming*
	Coffee break
	Roland Hildebrand: *On the convexity properties of the barrier parameter in conic programming*
	Bruno F. Lourenço: *On amenable cones and error bounds*
	Lunch
14:00	M. Seetharama Gowda: *Some structural properties of copositive and completely positive cones*
	José A. Samper: *Boundaries of completely positive* 5×5 *matrices*
	Coffee break
	Afonin Andrey: *The extreme rays of the* 6×6 *copositive cone*
	João Gouveia: *Cones of matrices of bounded factor width*
Evening	Christmas market in Linz city center

Talks Tuesday 10 December 2019

9:00	Miguel F. Anjos: *Tight-and-Cheap Conic Relaxations for Optimal Power Flow and Optimal Reactive Power Dispatch*
	Dang-Khoa Nguyen: *A Proximal-Gradient Algorithm for Nonconvex and Nonsmooth Optimization Problems with Applications to the Factorization of Completely Positive Matrices*
	Coffee break
	Felix Lieder: *Solving Large Scale Cubic Regularization by a Generalized Eigenvalue Problem*
	Christoph Helmberg: *Variable Metric in ConicBundle*
	Lunch
14:00	Paula Amaral: *Copositivity detection using DNN decomposition*
	Coffee break
	E. Alper Yıldırım: *On Doubly Nonnegative Relaxations of Standard Quadratic Programs*
	Angelika Wiegele: *Augmented Lagrangian Approaches for Solving Doubly Nonnegative Programs*
19:00	Conference dinner at Restaurant JOSEF, Landstraße 49, 4020 Linz. `https://www.josef.eu`

Talks Wednesday 11 December 2019

9:00	Frank Vallentin: *Complete positivity and distance-avoiding sets*
	Franz Rendl: *Lasserre Hierarchy versus Exact Subgraphs for Max-Cut and Stable-Set*
	Coffee break
	Renata Sotirov: *Facial reduction for symmetry reduced semidefinite programs*
	Jeffrey Zhang: *On Local Minima of Cubic Polynomials*
	Lunch
14:00	Luis F. Zuluaga: *Augmented Lagrangians for Polynomial Optimization Problems via Certificates of Non-negativity*
	Juan Vera: *Copositive Certificates of Non-negativity*
	Coffee and farewell

Abstracts

The extreme rays of the 6×6 copositive cone

Afonin Andrey
Moscow Institute of Physics and Technology, 9 Institutskiy per., Dolgoprudny, Moscow Region, 141700, Russian Federation

Abstract

We provide a complete classification of the extreme rays of the 6×6 copositive cone C^6. We proceed via a coarse intermediate classification of the possible minimal zero support set of an exceptional extremal matrix $A \in C^6$. To each such minimal zero support set we construct a union of manifolds in the space of real symmetric 6×6 matrices S^6, parameterized in a semi-trigonometric way, which consists of all exceptional extremal matrices $A \in C^6$ having this minimal zero support set.

Joint work with Roland Hildebrand, Univ. Grenoble Alpes, CNRS and Peter J.C. Dickinson, RaboBank.

Copositivity detection using DNN decomposition

Paula Amaral
D. Mathematics and CMA, FCT UNL, Portugal

Abstract

Copositivity plays an important role in optimization, particularly in discrete and quadratic optimization, since many of these problems admit a conic reformulation, or, at least, a relaxation over the completely positive cone and by duality we obtain a related conic problem over the copositive cone. Copositivity detection is difficult; in particular, to decide if it is not copositive is NP-complete. In this talk we present an algorithm that is based on the decomposition of a matrix in the doubly non-negative cone and present computational results for a set of Dimacs challenge instances.

This is a joint work with Immanuel Bomze, ISOR, VCOR, Data Science @ Uni Vienna, University of Vienna.

Tight-and-Cheap Conic Relaxations for Optimal Power Flow and Optimal Reactive Power Dispatch

Miguel F. Anjos
School of Mathematics, University of Edinburgh

Abstract

The classical alternating current optimal power flow (ACOPF) problem is highly nonconvex and generally hard to solve. Computational speed and global optimality are key needs for practical OPF algorithms. In practice, an OPF may be computed up to every few minutes to validate a market outcome or other operational aspects.

Convex relaxations of ACOPF, including conic, convex quadratic, and linear relaxations, have recently attracted significant interest. The semidefinite relaxation is the strongest among them and is exact for many cases. However, solving large-scale semidefinite optimization problems remains a challenge.

We present a conic optimization approach to ACOPF that combines semidefinite optimization with the reformulation-linearization technique (RLT) to obtain a Tight-and-Cheap

Relaxation (TCR) of ACOPF. TCR is tighter than the second-order cone relaxation and nearly as tight as the standard semidefinite relaxation. We show conditions under which TCR is exact and can provide a global optimal solution for the ACOPF problem, theoretically and computationally. Computational experiments using standard test cases with up to 6515 buses (nodes) show that the time to solve TCR is up to one order of magnitude lower than for the chordal relaxation, a semidefinite relaxation technique that exploits the sparsity of power networks.

We also consider the optimal reactive power dispatch (ORPD) problem. This is an extension of ACOPF where discrete control devices for regulating the reactive power, such as shunt elements and tap changers, are introduced. We model the ORPD problem as a mixed-integer nonlinear optimization problem, and apply the tight-and-cheap approach to it. We show that this relaxation, combined with a round-off technique, leads to near-global optimal solutions with very small optimality gaps. This is an improvement over the (nonconvex) continuous relaxation of ORPD. We report computational results on realistic test cases with up to 3375 buses.

Some power system applications that require solving OPFs are multi-period because of evolving factors such as market prices or demand behaviour. We propose a multi-period TCR for the multi-period ACOPF problem, and report computational experiments using test cases with up to 500 buses showing that this new relaxation is promising for power system applications.

This is joint work with Christian Bingane and Sébastien Le Digabel.

Cones of matrices of bounded factor width

João Gouveia
University of Coimbra

Abstract

In 2004, Boman et al introduced the concept of factor width of a semidefinite matrix A. This is the smallest k for which one can write the matrix as $A = VV^T$ with each column of V containing at most k non-zeros. The cones of such matrices provide natural relaxations to the semidefinite cones and, consequently, of the completely positive cones. Furthermore, in the polynomial optimization context, they can be used to check if a polynomial is a sum of squares of polynomials of support at most k.

In this talk, we will discuss some geometric properties of these cones, and show some applications. Namely to derive a relaxation for CP-programming and to prove some limitations on some recently proposed hierarchies for polynomial optimization.

Some structural properties of copositive and completely positive cones

M. Seetharama Gowda
University of Maryland, Baltimore County, USA

Abstract

Given a closed cone $\mathcal{C}$ that is not necessarily convex in $\mathbb{R}^n$, we consider two closed convex cones in the space $\mathcal{S}^n$ of $n \times n$ real symmetric matrices: the *completely positive cone of* $\mathcal{C}$ generated by $\{uu^T : u \in \mathcal{C}\}$ and its dual, the *copositive cone of* $\mathcal{C}$. These two cones appear prominently in conic programming.

In this talk, we describe some structural properties of these cones, focusing on irreducibility, homogeneity, Lyapunov/bilinearity transformations, and Lyapunov rank. Here, the Lyapunov rank (of a proper cone) is defined as the dimension of the space of all Lyapunov transformations on that cone or, equivalently, the dimension of the Lie algebra of the automorphism group of the cone. We illustrate these by considering $\mathcal{C} = \mathbb{R}^n$ which leads to the semidefinite cone whose Lyapunov rank is 'large' and $\mathcal{C} = \mathbb{R}^n_+$ which leads to the standard completely positive and copositive cones with 'small' Lyapunov rank.

Variable Metric in ConicBundle

Christoph Helmberg
TU Chemnitz

Abstract

For minimizing convex functions given via first order oracles, bundle methods form a cutting model by collecting subgradient information and optimize this model together with a proximal term that keeps solutions close to a current center of stability. In variable metric bundle methods this proximal term is adapted dynamically in order to include some second order information. In addition to the traditional polyhedral models the callable library ConicBundle offers special support for cutting models based on support functions over the semidefinite cone as suggested for the spectral bundle method. In this spectral setting it is quite well understood how to form a suitable proximal term in order to include some second order information. The computational cost incurred by the corresponding quadratic semidefinite subproblem is rather high. Therefore the current focus is on developing suitable techniques for increasing the efficiency of the subproblem solvers and this talk will report on these developments.

On the convexity properties of the barrier parameter in conic programming

Roland Hildebrand
Univ. Grenoble Alpes, CNRS, Grenoble INP, LJK, 38000 Grenoble, France

Abstract

Self-concordant barriers are a central object in interior-point methods for conic programming. Besides computability, the most important characteristic of the barrier is its parameter. The smaller the parameter, the faster the algorithms using this barrier converge. When considering conic programs over a given cone, it is therefore of interest to find barriers on this cone with a parameter as low as possible, i.e., to optimize the barrier parameter for the given cone.

It is, however, well-known that convex combinations of self-concordant functions do not need to be self-concordant. The aforementioned optimization problem is therefore non-convex, meaning that the sub-level set of barriers with parameter not exceeding a fixed value is not necessarily a convex set. In this contribution we investigate this non-convexity. We show that every function in the convex hull of the sub-level set corresponding to a value ν of the parameter can be scaled by multiplication with a positive constant to a self-concordant barrier with parameter $\tilde{\nu}$, where $\tilde{\nu} > \nu$ is an explicit function of ν.

This function can be described as follows. There exists a 1-parametric family $\{C_\nu\}_{\nu>2}$ of bodies in $\mathbb{R}^3$ such that $C_\nu \subset C_{\nu'}$ for every $\nu \leq \nu'$, and such that $\tilde{\nu}$ is the least value with the property that $C_{\tilde{\nu}}$ contains the convex hull of C_ν. We compute both C_ν and $\tilde{\nu}(\nu)$ explicitly. This result is tight, in the sense that $\tilde{\nu}$ cannot be improved, even for 2-dimensional cones.

The obtained result implies that convex relaxations of the problem of minimization of the barrier parameter lead to a performance loss which can be bounded in an explicitly quantifiable manner from above. For non-conic convex sets a similar result does not hold.

Solving Large Scale Cubic Regularization by a Generalized Eigenvalue Problem

Felix Lieder
Mathematisches Institut
Heinrich Heine Universität Düsseldorf

Abstract

In this talk we revisit the cubic regularization subproblem for both minimization and root finding of monotone operators. While cubic regularization methods have several favorable properties, their adoption among practitioners does not yet match the strong theoretical results. Two of the reasons for this discrepancy may be the additional implementation complexity as well as the higher computational cost of existing approaches. We first consider the unconstrained case and show that both of these obstacles can be avoided by reducing the subproblem to a generalized eigenvalue problem. The resulting algorithm is not only robust, due to existing highly advanced eigenvalue solvers, but also provides a new way of employing second order methods in the large scale case. In a second part the generalization of this approach to monotone operators is discussed. The application of this generalization to the "reduced Lagrangian" is aimed at solving large scale conic programs.

On amenable cones and error bounds

Bruno F. Lourenço
University of Tokyo

Abstract

The main topic of this talk is error bounds for conic linear systems. Usually, in order to obtain error bound results, it is necessary to assume constraint qualifications and/or regularity conditions. However, in 2000, Jos Sturm showed how error bounds for SDPs can be obtained without assuming regularity conditions. Furthermore, he showed that the quality of the bound depends on the singularity degree of the problem. The singularity degree, by its turn, is related to facial reduction, which is a general approach for regularizing conic optimization problems.

Taking Sturm's work as a starting point, we show that error bounds without constraint qualifications hold for a broad new family of cones called "amenable cones". Similarly, we show that the quality of the bound is also controlled by the singularity degree of the underlying problem. This highlights that facial reduction and error bounds are intrinsically connected, even in a relatively general setting. In particular, we provide a new Hölderian error bound for the doubly nonnegative cone and for symmetric cones, which recovers Sturm's result as a special case. At the end, if time allows, we will discuss a few algorithmic applications.

A Proximal-Gradient Algorithm for Nonconvex and Nonsmooth Optimization Problems with Applications to the Factorization of Completely Positive Matrices

Dang-Khoa Nguyen
University of Vienna

Abstract

We aim to factorize a completely positive matrix using an optimization approach that is based on an inertial proximal-gradient algorithm. The convergence analysis show that the iterates generated by this numerical scheme converge to a critical point of the objective function. Numerical experiments demonstrate the efficiency of the proposed method and emphasize the role played by the inertial step.

This is a joint work Radu Ioan Boţ (University of Vienna).

On positive duality gaps in semidefinite programming

Gábor Pataki
University of North Carolina at Chapel Hill

Abstract

We present a novel analysis of semidefinite programs (SDPs) with positive duality gaps, i.e., different optimal values in the primal and dual problems. These SDPs are considered extremely pathological, they are often unsolvable, and they also serve as models of more general pathological convex programs.

We first characterize two variable SDPs with positive gaps: we transform them into a standard form which makes the positive gap easy to recognize. The transformation is very simple, as it mostly uses elementary row operations coming from Gaussian elimination. We next show that the two variable case sheds light on larger SDPs with positive gaps: we present SDPs in any dimension in which the positive gap is certified by the same structure as in the two variable case. We analyze an important parameter, the *singularity degree* of the duals of our SDPs and show that it is the largest that can result in a positive gap.

We finally generate a library of difficult SDPs with positive gaps (some of these SDPs have only two variables), and a computational study.

Lasserre Hierarchy versus Exact Subgraphs for Max-Cut and Stable-Set

Franz Rendl
University of Klagenfurt, Austria

Abstract

Several hierarchies of semidefinite optimization relaxations for discrete problems were introduced in the last 30 years. The model of Lasserre (SIAM Journal on Optimization, 2002) is currently considered to be the strongest such hierarchy. Unfortunately, this hierarchy is not easily accessible from a computational point of view. Already the first nontrivial level of this hierarchy leads to semidefinite problems with matrices of order $\binom{n+1}{2}$ when starting from a binary problem in n variables. In contrast, the hierarchy based on exact subgraphs introduced by Adams et al (Information Systems and Operational Research 2015) has recently been shown to be quite an efficient tool to compute strong bounds for Max-Cut and Stable-Set (Gaar et al., Integer Programming and Combinatorial Optimization 2018). It operates on all levels in the space of $n \times n$ matrices.

In this talk we investigate how these two models relate to each other. It turns out that the exact subgraph hierarchy may be interpreted as a weak form of the Lasserre hierarchy. We investigate enhancements for the exact subgraph hierarchy which are derived from properties of the Lasserre hierarchy. We use the 'Semidefinite Matrix Completion Theory' based on chordal graphs to get a computational handle on the Lasserre hierarchy. As a result, we strengthen the exact subgraph model by introducing additional semidefiniteness constraints for matrices of small order to capture the power of the Lasserre hierarchy. Some preliminary computational experiments will be reported to demonstrate the potential of this approach.

Boundaries of completely positive 5×5 matrices

José A. Samper
Max Planck Institute for Mathematics in the Sciences

Abstract

We will discuss the geometry of the cone CP_5 of 5×5 completely positive matrices by using real algebraic geometry and numerics. Based on the description of the extreme rays of the copositive cone we classify all the semialgebraic components of the boundary part of CP_5 in the interior of the doubly non-negative cone. Such hypersurfaces have simple algebraic parametrisations that come in two types: low degree (dual to torus orbits of the Horn matrix) and high degree (dual to torus orbits of Hildebrandt matrices). Numerical algebraic geometry software tells us that the low degree hypersurfaces is 5 and the high degree hypersurfaces are of degree 320. The latter ones are therefore defined by a single polynomial that cannot be written down! We then propose a new numerical algorithm to factor matrices in the cone and test it using the parameterisations of the hypersurfaces above. If time permits we will also discuss the interior of the cone and the use of our algorithm to understand the transition between matrices of cp-rank 5 and 6.

This talk is based on joint work with Max Pfeffer.

Facial reduction for symmetry reduced semidefinite programs

Renata Sotirov
Tilburg University

Abstract

We consider the facial and symmetry reduction techniques for semidefinite programming. The combination of facial and symmetry reduction leads to a significant improvement in both numerical stability and running time for both the alternating direction method of multipliers and interior point method.

We test our method on various relaxations of hard combinatorial problems.

Parametric Analysis of Semidefinite and Second Order Conic Optimization

Tamás Terlaky
Department of Industrial and Systems Engineering, Lehigh University,
Behlehem, PS, USA

Abstract

Parametric analysis of optimization problems is crucial in understanding fundamental theoretical properties of optimization problems, and also for their application in engineering and business. While parametric analysis is well developed and understood for linear linearly constrained convex quadratic optimization problems, it is less developed for conic optimization, and the computation of the relevant quantities is considerably more challenging.

In this talk we study parametric analysis of SDO and SOCO problems w.r.t. the perturbation of the objective function. We study the behavior of the optimal partition and optimal set mapping in a so-called nonlinearity interval, and investigate the sensitivity of the approximation of the optimal partition in a nonlinearity interval. The approximation of the optimal partition is obtained from a bounded sequence of interior solutions on, or in a neighborhood of the central path. An upper bound on the distance between the approximations of the optimal partitions of the original and perturbed problems is presented.

Joint work with Ali Mohammad-Nezhad, Department of Mathematics, Purdue University, Lafayette, IN, USA; and Jonatha Hauenstein and Tingting Tang, Department of Applied and Computational Mathematics and Statistics, University of Notre Dame, IN, USA.

Complete positivity and distance-avoiding sets

Frank Vallentin
University of Cologne, Germany

Abstract

We introduce the cone of completely-positive functions, a subset of the cone of positive-type functions, and use it to fully characterize maximum-density distance-avoiding sets as the optimal solutions of a convex optimization problem. As a consequence of this characterization, it is possible to reprove and improve many results concerning distance-avoiding sets on the sphere and in Euclidean space.

Based on joint work (arXiv:1804.09099 [math.MG]) with Evan DeCorte and Fernando Mário de Oliveira Filho.

Copositive Certificates of Non-negativity

Juan Vera
Tilburg University

Abstract

A certificate of non-negativity is a way to write a given function so that its non-negativity becomes evident. Classical non-negativity certificates such as Schmudgen's and Putinar's Positivstellensatz make the non-negativity of a polynomial over a semialgebraic set evident using sums of squares (sos) polynomials. Recently, both for theoretical and practical reasons, alternatives to the use of sos polynomials have been considered to construct certificates of non-negativity. In particular, circuit, scaled diagonally dominant, hyperbolic, SAGE, and copositive polynomials have been proposed for this purpose.

In this talk, we discuss theoretical and practical implications of using copositive certificates of non-negativity. In particular we show that copositive polynomials can be used to obtain novel structured/sparse certificates of non-negativity, as well as novel certificates of non-negativity using circuit, hyperbolic, and other classes polynomials. Also, we will show how copositive certificates of non-negativity and some of the key ideas behind their construction, can be used to obtain analysis-based proofs of the existence of classical certificates of non-negativity.

Join work with Olga Kuryatnikova and Luis Zuluaga.

Augmented Lagrangian Approaches for Solving Doubly Nonnegative Programs

Angelika Wiegele
Alpen-Adria-Universität Klagenfurt

Abstract

Augmented Lagrangian methods are among the most popular first-order approaches to handle large scale semidefinite programs (SDP). Alternating direction method of multipliers (ADMM) is a variant of the augmented Lagrangian scheme that performs consecutive updates on certain blocks of the dual variables.

We present an ADMM where we investigate the possibility of eliminating the positive semidefinite constraint on the dual matrix by employing a factorization. A description on how to deal with the resulting unconstrained maximization of the augmented Lagrangian is given.

In particular, we look at doubly nonnegative programs (DNN), these are semidefinite programs where the elements of the matrix variable are constrained to be nonnegative. In an ADMM, these constraints can be handled efficiently since this simply amounts in projections onto the non-negative orthant.

Whenever the SDP or DNN is a relaxation of a combinatorial optimization problem, one is interested in using its solution within a branch-and-bound framework. ADMM typically does not yield solutions with a high precision. Having solutions of moderate precision only, the obtained optimum of the SDP or DNN might not be a valid bound for the underlying combinatorial optimization problem. We present a post-processing to obtain valid bounds from the solution of the ADMM. This valid bound can be computed at low cost and the numerical results demonstrate that this procedure does not weaken the bound significantly.

This is joint work with Martina Cerulli, Marianna De Santis, Elisabeth Gaar and Franz Rendl.

On Doubly Nonnegative Relaxations of Standard Quadratic Programs

E. Alper Yıldırım
University of Edinburgh

Abstract

A standard quadratic program (StQP) is an optimization problem in which a (nonconvex) quadratic form is minimized over the unit simplex. We focus on doubly nonnegative relaxations obtained from copositive programming formulations of standard quadratic programs. We establish several structural properties of instances of standard quadratic programs that admit an exact doubly nonnegative relaxation. We identify several classes of such standard quadratic programs. We also discuss procedures for generating instances of standard quadratic programs with an exact doubly nonnegative relaxation as well as instances with a positive gap.

This is joint work with Yakup Gorkem Gokmen.

On Local Minima of Cubic Polynomials

Jeffrey Zhang
Princeton University

Abstract

We study local minima of cubic polynomials. We first give a characterization of local minima, and show that this characterization can be checked in polynomial time. We then study the geometry of local minima of cubic polynomials, and that detecting whether a cubic polynomial has any local minima is as hard as SDP feasibility. Nonetheless, we give an SDP approach for finding local minima of cubic polynomials.

Augmented Lagrangians for Polynomial Optimization Problems via Certificates of Non-negativity

Luis F. Zuluaga
Department of Industrial and Systems Engineering, Lehigh University

Abstract

The Lagrangian function associated with an optimization problem has and continuous to be of the utmost importance to develop algorithmic solution approaches for optimization problems. This is particularly the case when the problem of interest is convex, as in that case, optimizing the Lagrangian function provides a dual problem (Lagrangian dual) to the primal (original) one, satisfying strong duality. For non-convex problems, an augmented Lagrangian function, including non-linear penalty functions on the problem constraints, has been studied in the literature to address the solution of non-convex problems using Lagrangian relaxation solution

methods. In this talk, we show how augmented Lagrangian formulations for polynomial optimization (PO) problems satisfying strong duality can be obtained using PO techniques, and in particular, using a special class of structured certificates of non-negativity. In the special case of PO problems over the non-negative orthant, this approach can be used to show that the well-known completely positive reformulation of the PO satisfies strong duality with respect to its associated conic dual problem over the cone of copositive polynomials. Furthermore, for other particular classes of PO problems one can show that exact augmented Lagrangian reformulations can be obtained, that is, an augmented Lagrangian formulation in which the multipliers associated with the penalty terms can be bounded or computed explicitly. This joint work with Juan. C. Vera, Tilburg School of Economics and Management, Tilburg University.