RICAM Special Semester on Optimization

Workshop 4
Nonsmooth Optimization

Book of Abstracts

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Linz, Austria
November 25–27, 2019

The Boosted DC Algorithm for linearly constrained DC programming

Francisco J. Aragón Artacho
Department of Mathematics, University of Alicante, Alicante, Spain

Abstract

The Boosted Difference of Convex functions Algorithm (BDCA) has been recently introduced to accelerate the performance of the classical Difference of Convex functions Algorithm (DCA). This acceleration is archived thanks to an extrapolation step from the point computed by DCA via a line search procedure. In addition, empirical results have shown that BDCA has better chances to escape from bad local optima toward solutions with a better objective value than the classical DCA. In this talk we will show how to extend BDCA to solve a class of DC programs with linear constraints. We propose a new variant of BDCA and establish its global convergence to a critical point. Finally, we present some numerical experiments demonstrating that this new variant of BDCA outperforms DCA both in running time and objective value of the solutions obtained.

This is a joint work with R. Campoy, Q. Tran-Dinh and P. T. Vuong.

On inner calmness, generalized calculus, and derivatives of the normal-cone map*

Matúš Benko
Institute of Computational Mathematics, Johannes Kepler University Linz

Abstract

In this talk, we discuss the inner-type continuity and Lipschitzian properties of set-valued maps. We introduce the new notion of inner calmness* and show its remarkable features, in particular that it is satisfied by polyhedral maps. Then we utilize these inner-type conditions to extend the known and to build new generalized differential calculus rules, focusing on the primal objects (e.g. tangent cones). The exact chain rule for the graphical derivatives deserves the special attention. Finally, we apply these results to compute or estimate the generalized derivatives of the so-called normal cone mapping, that are critical for stability analysis, etc. As a specific application, we show some interesting results regarding the newly developed property of semismoothness* of the normal cone maps.

A primal-dual dynamical approach to structured convex minimization problems

Radu Ioan Boţ
Faculty of Mathematics, University of Vienna

Abstract

We present a primal-dual dynamical approach to the minimization of a structured convex function consisting of a smooth term, a nonsmooth term, and the composition of another nonsmooth term with a linear continuous operator. We introduce a dynamical system for which we prove that its trajectories asymptotically converge to a saddle point of the Lagrangian of the underlying convex minimization problem as time tends to infinity. In addition, we provide rates for both the violation of the feasibility condition by the ergodic trajectories and the convergence of the objective function along these ergodic trajectories to its minimal value. Explicit time discretization of the dynamical system results in a numerical algorithm which is a combination of the linearized proximal method of multipliers and the proximal ADMM algorithm.

Primal-dual splittings as fixed point iterations in the range of linear operators

Luis Briceño-Arias
Department of Mathematics, Universidad Técnica Federico Santa María

Abstract

In this talk we study fixed point iterations of quasinonexpansive mappings in the range of monotone self-adjoint linear operators, which defines a real Hilbert space. This setting appears naturally in primal-dual algorithms for solving composite monotone inclusions including, as a particular instance, the Douglas–Rachford splitting. We first study conditions under which the range of a monotone self-adjoint linear operator endowed with the corresponding positive semidefinite inner product defines a Hilbert subspace, generalizing the non-standard metric case in which the linear operator is coercive and its range is the whole space. Next we study the convergence of fixed point iterations in this Hilbert subspace as a shadow of iterates in the whole Hilbert space. The result is applied to obtain the convergence of primal-dual splittings for critical values of stepsizes, generalizing results obtained in Condat (2013). We study in detail the case of the Douglas–Rachford splitting, which is first interpreted as a primal-dual algorithm with critical stepsize values whose associated operator is firmly nonexpansive in a Hilbert subspace. A second primal-dual interpretation is provided with an alternative operator which is firmly quasinonexpansive in the whole primal-dual space with a non-standard metric. We thus obtain the weak convergence of primal-dual shadow sequences as in Svaiter (2010). We finish with some numerical experiences and applications in image processing.

This is a joint work with Fernando Roldán.

No-gap second-order optimality conditions for bang-bang optimal control problems via second subderivatives

Constantin Christof
Center for Mathematical Sciences, M17, Technische Universität München

Abstract

This talk is concerned with necessary and sufficient second-order optimality conditions for constrained optimization problems that are posed in the dual of a separable Banach space. Using a suitably defined notion of second subderivative for the characteristic function of the admissible set, we derive no-gap second-order optimality conditions in an abstract functional analytic setting. The presented theory not only covers those cases where the classical assumptions of polyhedricity or second-order regularity are satisfied but also allows to study problems in the absence of these requirements. As a tangible example, we consider bang-bang optimal control problems governed by semi-linear partial differential equations. For this problem class, we obtain no-gap second-order optimality conditions that allow to identify precisely how the curvature of the admissible set contributes to the quadratic growth of the objective function.

The talk is based on joint work with Gerd Wachsmuth.

Approaches to bilevel optimization

Stephan Dempe
Department of Mathematics and Computer Science, TU Bergakademie Freiberg

Abstract

The topic of bilevel optimization is the minimization of some function subject to a subset of the graph of the solution set mapping of a second optimization problem. While being formulated, say, using differentiable functions, it is an optimization problem with a nonconvex, nonsmooth feasible set. In the presentation, we first present different approaches to transform it into a (nonsmooth) single-level optimization problem. In the second part, the transformation using the optimal value function of the second problem is considered in more detail. Under certain assumptions, the approximation of this function provides an encouraging approach for solving the bilevel optimization problem.

Nonsmoothness in the context of probability functions

René Henrion
Weierstrass Institute Berlin, Germany

Abstract

Probability functions play an important role in stochastic optimization and asscoiated engineering applications (reliability maximization, probabilistic constraints). They refer to a parametric system of random inequalities, where the parameter is the decision variable of some optimization problem. Then, the probability function assigns to the value of the parameter the probability of satisfying the given inequality system. Even if all input data (inequalities, distribution of the random vector) are smooth, the same is not necessarily true for the probability function. The talk presents a methodology of applying tools from generalized differentiation in order to characterize subdifferentials of probability functions and, more imprtantly, to identify appropriate additional conditions finally ensuring smoothness and providing explicit gradient formulae which can then be applied to the algorithmic solution of such optimization problems.

A study of convex convex-composite functions

Tim Hoheisel
Department of Mathematics and Statistics, McGill

Abstract

In this talk we present a full conjugacy and subdifferential calculus for convex convex-composite functions in finite-dimensional space. Our approach, based on infimal convolution and cone-convexity, is straightforward and yields the desired results under a verifiable Slater-type condition, with relaxed monotonicity and without lower semicontinuity assumptions on the functions in play. The versatility of our findings is illustrated by a series of applications in optimization and matrix analysis, including conic programming, matrix-fractional, variational Gram, and spectral functions.

Quasi-variational inequalities in Banach spaces: existence result and augmented Lagrangian method

Christian Kanzow
University of Würzburg

Abstract

This paper deals with quasi-variational inequality problems (QVIs) in a generic Banach space setting. We provide a theoretical framework for the analysis of such problems which is based on two key properties: the pseudomonotonicity (in the sense of Brezis) of the variational operator and a Mosco-type continuity of the feasible set mapping. We show that these assumptions can be used to establish the existence of solutions and their computability via suitable approximation techniques.

Based on the theoretical framework, we construct an algorithm of augmented Lagrangian type which reduces the QVI to a sequence of standard variational inequalities. A full convergence analysis is provided which includes the existence of solutions of the subproblems as well as the attainment of feasibility and optimality. Applications and numerical results are included to demonstrate the practical viability of the method.

This talk is based on joint work with Daniel Steck.

Inconsistent Nonconvex Feasibility: algorithms and quantitative convergence results

Russell Luke
Universität Göttingen

Abstract

Feasibility models can be found everywhere in optimization and involve finding a point in the intersection of a collection (possibly infinite) of sets. The focus of most of the theory for feasibility models is on the case where the intersection is nonempty. But in applications, one encounters infeasible problems more often than might be expected. We examine a few high-profile instances of inconsistent feasibility and demonstrate the application of a recently established theoretical framework for proving convergence, with rates, of some elementary algorithms for inconsistent feasibility.

Scaled improvement functions in nonsmooth multiobjective bundle methods

Marko M. Mäkelä
Department of Mathematics and Statistics, University of Turku, Finland

Abstract

Improvement functions are used in nonsmooth optimization both for constraint handling and scalarization of multiple objectives. In the multiobjective case the improvement function possesses, for example the nice property that a descent direction for the improvement function improves all the objectives of the original problem. However, the numerical experiments have shown that the standard improvement function is rather sensitive for scaling. For this reason we present here a new scaled version of the improvement function capable not only for linear but also for polynomial, logarithmic, and exponential scaling for both objective and constraint functions. In order to be convinced about the usability of the scaled improvement function, we develop a new version of the multiobjective proximal bundle method utilizing the scaled improvement function. This new method can be proved to produce weakly Pareto stationary solutions. In addition, under some generalized convexity assumptions the solutions are guaranteed to be globally weakly Pareto optimal. Furthermore, we illustrate the affect of the scaling with some numerical examples.

This is a joint work with Outi Montonen.

A globally and locally fast convergent semismooth quasi-Newton method for equations with nonsmoothness of Fischer–Burmeister type

Florian Mannel
University of Graz

Abstract

We present a nonmonotone line search globalization of semismooth Newton methods and semismooth quasi-Newton methods. Here, the nonmonotonicity allows to design methods that always use the Newton direction. Under suitable assumptions this approach results in global and fast local convergence for finite-dimensional nonsmooth equations whose nonsmoothness is given by Fischer–Burmeister-type complementarity functions. The methods apply to, e.g., nonlinear complementarity problems, generalized Nash equilibrium problems, and constrained nonlinear optimization problems. We discuss some of the applications and provide preliminary numerical results.

This is joint work with Karl Kunisch.

Viscous approximation of optimization problems constrained by rate-independent systems with non-convex energies

Christian Meyer
TU Dortmund

Abstract

We consider an optimization problem governed by a rate-independent system of the form

$$0 \in \mathcal{R}(\dot{z}) + D_z \mathcal{I}(\ell, z), \tag{1}$$

where $\mathcal{R}$ is a convex and positively homogeneous dissipation functional and $\mathcal{I}$ denotes a smooth, but non-convex energy. Moreover, ℓ is the driving force of the system and z denotes its state. It is well known that, in case of a non-convex energy, a rate-independent system of this form does in general not admit a unique differential solution. Therefore, several alternative solution concepts have been developed, among them the so-called parametrized solutions. We consider an optimization problem equipped with this notion of solutions. Beside the question of existence of globally optimal solutions, we are interested in the approximation of global minimizers by means of viscous regularization, where a viscous term of the form $\varepsilon \dot{z}$ is added to the right hand side of (1). The viscous system arising in this way is uniquely solvable and its solution is Lipschitz continuous in time. In order to show that globally optimal solutions of the viscous regularized optimization problem converge to global minimizers of the original problem for $\varepsilon \searrow 0$, one needs to verify the so-called reverse approximation property, i.e., a recovery sequence for a global minimizer has to be constructed. If the underlying space is finite dimensional, this may be done by a suitable penalization technique.

This is joint work with Dorothee Knees (Kassel) and Michael Sievers (TU Dortmund).

Parabolic regularity in variational analysis and applications

Boris Mordukhovich
Wayne State University, Detroit, USA

Abstract

The talk is devoted to systematic developments and applications of geometric aspects of second-order variational analysis and optimization that are revolved around the concept of parabolic regularity of sets. This concept has been known in variational analysis for more than two decades while being largely underinvestigated. We discover here that parabolic regularity is the key to derive new calculus rules and computation formulas for major second-order generalized differential constructions of variational analysis. The established results of second-order variational analysis and generalized differentiation, being married to the developed calculus of parabolic regularity, allow us to obtain novel applications to both qualitative and algorithmic aspects of constrained optimization including second-order optimality conditions, augmented Lagrangians, SQP and Newton-type methods, etc.

Based on joint work with Ashkan Mohammadi (WSU) and Ebrahim Sarabi (Miami University).

Multiobjective optimal control of a non-smooth semi-linear elliptic PDE

Georg Müller
University of Konstanz

Abstract

Scalar optimization problems with non-smooth PDEs have been researched considerably over the last years. When optimal compromises (i.e. Pareto optimal points) for optimization problems with multiple objectives and non-smooth PDE constraints are sought after, only few results are known. This talk addresses the multiobjective optimal control of a non-smooth semi-linear elliptic PDE with max-type nonlinearity. The presentation covers existence of Pareto optimal points, C- and strong stationarity conditions in the multiobjective setting as well as corresponding numerical results for examples with up to 3 cost functionals.

This is joint work with Constantin Christof (TU Munich).

On the semismooth Newton method and its application to a class of Nash equilibria*

Jiří V. Outrata
Institute of Information Theory and Automation, The Czech Academy of Sciences

Abstract

On the basis of the concept of semismothness* a new Newton-type method is derived which is applicable to the numerical solution of generalized equations (GEs). If the right-hand side of the considered GE is strongly metrically regular around the solution, then in the Newton step one may employ the B-subdifferential of the inverse mapping. This method is applied to a GE which governs Nash equilibria in presence of nonsmooth terms in the objectives of the single players.

This is a joint research with H. Gfrerer (Linz).

Stability of the solution set of quasi-variational inequalities

Carlos N. Rautenberg
George Mason University, Fairfax, VA, USA

Abstract

For obstacle-type quasivariational inequalities (QVIs), we consider the stability of its solution set and associated optimal control problems. The latter are non-standard optimization problems in the sense that they involve an objective with set-valued arguments. The approach to study the solution stability is based on perturbations of minimal and maximal elements to the solution set of the QVI with respect to monotonic perturbations of the forcing term. It is shown that different assumptions are required for studying decreasing and increasing perturbations and that the optimization problem of interest is well-posed.

An inexact bundle method for nonconvex nonsmooth minimization in Hilbert spaces with applications to optimal control of elliptic variational inequalities

Michael Ulbrich
Technical University of Munich

Abstract

Motivated by optimal control problems for elliptic variational inequalities we develop an inexact bundle method for nonsmooth nonconvex minimization subject to general convex constraints. The proposed method requires only approximate (i.e., inexact) evaluations of the cost function and of an element of the generalized differential. The algorithm allows for incorporating curvature information while aggregation techniques ensure that an approximate solution of the piecewise quadratic subproblem can be obtained efficiently. Beyond inexact versions of Clarke's subdifferential we discuss minimum requirements for suitable approximate subdifferentials and explore ways to weaken approximate convexity (or lower C^1) assumptions. A global convergence theory in a suitable infinite-dimensional Hilbert space setting is presented. We study the application of our framework to the optimal control of deterministic and stochastic obstacle problems and present numerical results. If time permits, we also plan to address adaptive inexactness control based on error estimates for the cost function and for the solutions of the bundle subproblems.

This is joint work with Lukas Hertlein. The work is funded by the DFG within the SPP 1962.

On computation of optimal strategies in oligopolistic markets respecting the cost of change

Jan Valdman
Institute of Information Theory and Automation, The Czech Academy of Sciences

Abstract

We investigate optimal strategies of producers in the oligopolistic markets, where the so called *costs of change* are taken into account. It is shown that a variant of the Gauss-Seidel method, proposed for the computation of Nash equilibria, can be applied. Further, we examine the two-level case, when one firm decides to take over the role of the Leader (Stackelberg equilibrium). The results are illustrated via an academic example.

This is a joint research with Jiří V. Outrata (Prague).

First-order primal-dual methods for non-linear inverse problems

Tuomo Valkonen
Escuela Politécnica Nacional & University of Helsinki

Abstract

Convex optimisation problems can frequently be solved more efficiently by converting their original primal form into a dual form or saddle-point form. A popular algorithm for such problems is the primal–dual proximal splitting (PDPS) of Chambolle and Pock. Until recently, non-convex problems were most commonly solved by second-order methods in their primal form. In this talk, we discuss recent extensions of the PDPS to increasingly more complex non-smooth non-convex optimisation problems arising from non-linear inverse problems

Parallel random block-coordinate forward-backward algorithm: a unified convergence analysis

Silvia Villa
Dipartimento di Matematica, Università di Genova

Abstract

I will present a block coordinate forward-backward algorithm where the blocks are updated in a random and possibly parallel manner, according to arbitrary probabilities. The algorithm allows different stepsizes along the block-coordinates to fully exploit the smoothness properties of the objective function. I will consider the convex case and provide a unifying analysis of the convergence under different hypotheses, advancing the state of the art under several aspects.

Proximal gradient method applied to optimization problems with L^p-cost, $p \in [0, 1)$

Daniel Wachsmuth
University of Würzburg

Abstract

We are interested in optimization problems of the type

$$\min f(u) + \frac{\alpha}{2}\|u\|^2_{L^2(\Omega)} + \beta \int_\Omega |u|^p \, dx,$$

where $\Omega \subset \mathbb{R}^n$ is bounded, $f : L^2(\Omega) \to \mathbb{R}$ is smooth. In order to obtain approximate solutions of this problem, we consider the following proximal gradient iteration: Given u_k, find u_{k+1} as solution of

$$\min f(u_k) + \nabla f(u_k)(u - u_k) + \frac{L}{2}\|u - u_k\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|u\|^2_{L^2(\Omega)} + \beta \int_\Omega |u|^p \, dx.$$

Here, $L > 0$ can be chosen a-priori or determined by line-search. We investigate the convergence properties of the sequence of iterates. Under mild assumptions, iterates converge strongly for $p = 0$. In addition, we investigate properties of weak limit points of the sequence of iterates. In general, they satisfy conditions that are weaker than the Pontryagin maximum principle.