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Integers in Stochastic Optimization

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The Classic Case and Beyond :

Continuous Variables – Convexity !!

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Minor modification with major impact:

$$\Phi(t) = \min\left\{\frac{1}{2}v + y^+ + y^- \ : \ v + y^+ - y^- = t,\ v \in \mathbb{Z}_+,\ y^+ \in \boldsymbol{R}_+, y^- \in \boldsymbol{R}_+\right\}$$

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The Classic Case and Beyond : Continuous Variables – Convexity !!

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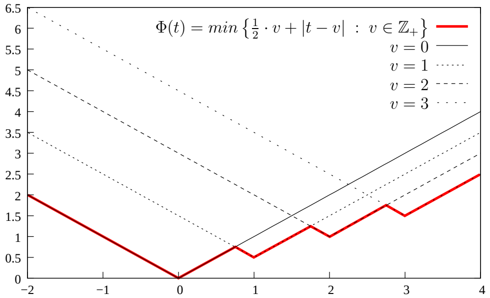

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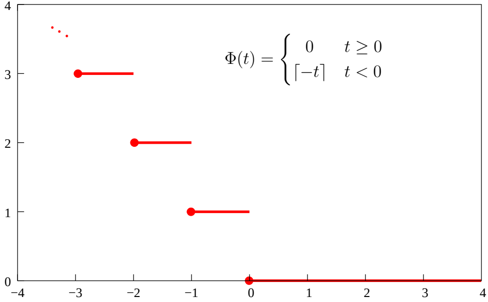

Random linear (mixed-integer) program:

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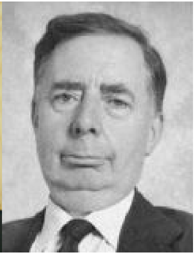
Stochastic programmng is about "optimizing or ranking against each other the random variables $f(x,.)$ as x varies".

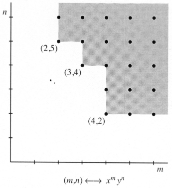
Stochastic Integer Programs:
Hilbert, Graver, Gordan-Dickson, and Maclagan

with Raymond Hemmecke:

Decomposition of test sets in stochastic integer programming,
Mathematical Programming 94 (2003), 323 - 341.



Gordan+Dickson →deterministic, integer vectors
Maclagan → two-stage stochastic, monomial ideals
Nash-Williams, Aschenbrenner+Hemmecke → multi-stage stoch., vector trees

Issues – IP:

- **Ideal:** $\mathcal{I} \subseteq k[x_1, \ldots, x_n]$ ideal, if
  (i) $0 \in \mathcal{I}$;
  (ii) If $f, g \in \mathcal{I}$, then $f + g \in \mathcal{I}$;
  (iii) If $f \in \mathcal{I}$ and $h \in k[x]$, then $hf \in \mathcal{I}$
- **Ground Set:** $\mathcal{S} := \mathbb{Z}^n$
- **Partial Order on $\mathbb{Z}^n$:** $u \sqsubseteq v$, if

  $$|u^{(j)} \cdot v^{(j)} \geq 0 \quad \text{and} \quad |u^{(j)}| \leq |v^{(j)}| \quad \text{for all components } j.$$

  Commonly said "u reduces v"
- **The Set B**

Optimality Certificates, Test Sets

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Definition (Optimality Certificate, Test Set)
A set $\mathcal{T}_c \subseteq \mathbb{Z}^d$ is called an optimality certificate (or test set) for the family of problems

$$(IP)_{c,b} \qquad \min\{c^\intercal z : Az = b,\, z \in \mathbb{Z}_+^d\}$$

as $b \in \mathbb{R}^l$ varies if

1. $c^\intercal t > 0$ for all $t \in \mathcal{T}_c$, and
2. for every $b \in \mathbb{R}^l$ and for every non-optimal feasible solution $z_0 \in \mathbb{Z}_+^d$ to $Az = b$, there exists an improving vector $t \in \mathcal{T}_c$ such that $z_0 - t$ is feasible.

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A set $\mathcal{T}$ is called a universal optimality certificate for the family of problems $(IP)_{c,b}$ as $b \in \mathbb{R}^l$ and $c \in \mathbb{R}^d$ vary if it contains an optimality certificate $\mathcal{T}_c$ for every $c \in \mathbb{R}^d$.

Augmentation, Feasibility

Algorithm (Augmentation Algorithm)

Input: a feasible solution z_0 to $(IP)_{c,b}$, an optimality certificate $\mathcal{T}_c$ for $(IP)_{c,b}$

Output: an optimal point $z_{\min}$ of $(IP)_{c,b}$

while there is $t \in \mathcal{T}_c$ with $c^\intercal t > 0$ such that $z_0 - t$ is feasible do

$z_0 := z_0 - t$

return z_0

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return z_0

Algorithm (Feasible Solution)

Input: a solution $z_1 \in \mathbb{Z}^d$ to $Az = b$, a universal optimality certificate $\mathcal{T}$ for $(IP)_{c,b}$
Output: a feasible solution to $(IP)_{c,b}$ or "FAIL" if no such exists

While there is some $g \in \mathcal{T}$ such that $g \leq z_1^+$ and $\|(z_1 - g)^-\|_1 < \|z_1^-\|_1$ do
$z_1 := z_1 - g$
if $\|z_1^-\|_1 > 0$ then return "FAIL" else return z_1

Definition (Hilbert basis)
Let C be a polyhedral cone with rational generators. A finite set $H = \{h_1, \ldots, h_t\} \subseteq C \cap \mathbb{Z}^d$ is a Hilbert basis of C if every $z \in C \cap \mathbb{Z}^d$ has a representation of the form

$$z = \sum_{i=1}^{t} \lambda_i h_i,$$

with non-negative integral multipliers $\lambda_1, \ldots, \lambda_t$.

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Let $\mathbb{O}_j$ be the j^{th} orthant of $\mathbb{Z}^d$ and $H_j(A)$ be the unique minimal Hilbert basis of the pointed rational cone $\{v \in \mathbb{R}^d : Av = 0\} \cap \mathbb{O}_j$.

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Lemma (Graver set)
The set

$$\mathcal{G}(A) := \bigcup H_j(A) \setminus \{0\}$$

is a universal optimality criterion, called the IP Graver set or IP Graver basis, for the family of problems $(IP)_{c,b}$ as $b \in \mathbb{R}^l$ and $c \in \mathbb{R}^d$ vary.

$u \sqsubseteq v$ iff

- $u^+ \leq v^+$ and $u^- \leq v^-$ where $\mathsf{max}\{0, u^{(i)}\}$ are the components of u^+ and $\mathsf{max}\{0, -u^{(i)}\}$ those of u^-.

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Algorithm (Normal Form Algorithm)

Input: a vector s, a set G of vectors
Output: a normal form of s with respect to G

while there is some $g \in G$ such that $g \sqsubseteq s$ do
    $s := s - g$
return s

Algorithm (Computing IP Graver Sets)

Input: $F = \bigcup_{f \in F(A)} \{f, -f\}$, where $F(A)$ is a set of vectors generating $\ker(A)$ over $\mathbb{Z}$

Output: a set G which contains the IP Graver set $\mathcal{G}(A)$

$G := F$

$C := \bigcup_{f,g \in G} \{f + g\}$ (forming S-vectors)

while $C \neq \emptyset$ do

  $s :=$ an element in C

  $C := C \setminus \{s\}$

  $f :=$ normalForm(s, G)

  if $f \neq 0$ then

    $C := C \cup \bigcup_{g \in G} \{f + g\}$ (adding S-vectors)

    $G := G \cup \{f\}$

return G.

Two-Stage Stochastic Integer Programs

$$\min\{c^\intercal z : A_N z = b, z \in \mathbb{Z}_+^d\}$$

$$A_N := \begin{pmatrix} A & 0 & 0 & \cdots & 0 \\ T & W & 0 & \cdots & 0 \\ T & 0 & W & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ T & 0 & 0 & \cdots & W \end{pmatrix}$$

with

N denoting the number of scenarios, $d = m + Nn$,

$c = (c_0, c_1, \ldots, c_N)^\intercal := (h, \pi_1 q, \ldots, \pi_N q)^\intercal$

$b = (a, \xi^1, \ldots, \xi^N)^\intercal$.

We will

- extend the notion of S-vectors
- retain the pattern of the previous completion method
- work on pairs (u, V_u) (defined below) instead of vectors.
- employ a generalization of the Gordan-Dickson Lemma, Maclagan's Theorem (Proc. AMS, 2001), to ensure termination of the algorithm.
- see, that the block angular structure of the problem matrix induces a symmetry structure on the elements of the Graver set.
- see, that the Graver set vectors are formed by a comparably small number of building blocks.
- compute these building blocks without computing the Graver set.
- reconstruct an improving vector to a given non-optimal feasible solution, scenario by scenario, using building blocks only.
- find an optimal solution with comparably small effort, once the building blocks have been computed.

Lemma $(u, v_1, \ldots, v_N) \in \ker(A_N)$ if and only if $(u, v_1), \ldots, (u, v_N) \in \ker(A_1)$.

Definition
Let $z = (u, v_1, \ldots, v_N) \in \ker(A_N)$ and call the vectors $u, v_1, \ldots, v_N$ the building blocks of z. Denote by $\mathcal{G}_N$ the Graver test set associated with A_N and collect into $\mathcal{H}_N$ all those vectors arising as building blocks of some $z \in \mathcal{G}_N$. By $\mathcal{H}_\infty$ denote the set $\bigcup_{N=1}^{\infty} \mathcal{H}_N$.

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The set $\mathcal{H}_\infty$ contains both m-dimensional vectors u associated with the first-stage and n-dimensional vectors v related to the second-stage in the stochastic program. For convenience, we will arrange the vectors in $\mathcal{H}_\infty$ into pairs (u, V_u).

Definition
For fixed $u \in \mathcal{H}_\infty$, all those vectors $v \in \mathcal{H}_\infty$ are collected into V_u for which $(u, v) \in \ker(A_1)$.

Finiteness of $\mathcal{H}_\infty$

Definition
We say that $(u', V_{u'})$ reduces (u, V_u), or $(u', V_{u'}) \sqsubseteq (u, V_u)$ for short, if the following conditions are satisfied:

- $u' \sqsubseteq u$,
- for every $v \in V_u$ there exists a $v' \in V_{u'}$ with $v' \sqsubseteq v$,
- $u' \neq 0$ or there exist vectors $v \in V_u$ and $v' \in V_{u'}$ with $0 \neq v' \sqsubseteq v$.

Theorem (Maclagan 2001)
Let $\mathcal{I}$ be an infinite collection of monomial ideals in a polynomial ring. Then there are two ideals $I, J \in \mathcal{I}$ with $I \subseteq J$.

Definition
We associate with (u, V_u), $u \neq 0$,and with $(0, V_0)$ the monomial ideals

$$I(u, V_u) \in Q[x_1, \ldots, x_{2m+2n}] \qquad \text{and} \qquad I(0, V_0) \in Q[x_1, \ldots, x_{2n}]$$

generated by all the monomials $x^{(u^+,u^-,v^+,v^-)}$ with $v \in V_u$, and by all the monomials $x^{(v^+,v^-)}$ with $v \neq 0$ and $v \in V_0$, respectively.

Lemma
Let $((u_1, V_{u_1}), (u_2, V_{u_2}), \ldots)$ be a sequence of pairs such that $(u_i, V_{u_i}) \not\sqsubseteq (u_j, V_{u_j})$ whenever $i < j$. Then this sequence is finite.

Theorem (Finiteness of H_∞)
Given rational matrices A, T, and W of appropriate dimensions, and let $\mathcal{H}_\infty$ be defined as above. Then $\mathcal{H}_\infty$ is a finite set.

Computation of $\mathcal{H}_\infty$

Idea:

- Retain the completion pattern of Graver set computation, but work with pairs (u, V_u) instead.
- Define the two main ingredients, S-vectors and normalForm, that means the operations $\oplus$ and $\ominus$, appropriately.
- Now, the objects f, g, and s all are pairs of the form (u, V_u).

Algorithm (Extended normal form algorithm)

Input: a pair s, a set G of pairs
Output: a normal form of s with respect to G

while there is some $g \in G$ such that $g \sqsubseteq s$ do $s := s \ominus g$
return s

Algorithm (Compute $\mathcal{H}_\infty$)

Input: a generating set F of $\ker(A_1)$ in (u, V_u)-notation to be specified below

Output: a set G which contains $\mathcal{H}_\infty$

$G := F$

$C := \bigcup_{f,g \in G} \{f \oplus g\}$ (forming S-vectors)

while $C \neq \emptyset$ do

    $s :=$ an element in C

    $C := C \setminus \{s\}$

    $f := \text{normalForm}(s, G)$

    if $f \neq (0, \{0\})$ then

        $C := C \cup \bigcup_{g \in G \cup \{f\}} \{f \oplus g\}$ (adding S-vectors)

        $G := G \cup \{f\}$

return G.

Choose as input the set of building blocks of all vectors in $F \cup \{0\}$ in (u, V_u)-notation. Herein, F is a generating set for $\ker(A_1)$ over $\mathbb{Z}$ which contains a generating set for

$$\{(0, v) : Wv = 0\} \subseteq \ker(A_1)$$

consisting only of vectors with zero first-stage component.

Definition (S-vectors, Reduction)
Let

$$(u, V_u) \oplus (u', V_{u'}) := (u + u', V_u + V_{u'}),$$

where

$$V_u + V_{u'} := \{v + v' : v \in V_u, v' \in V_{u'}\}.$$

Moreover, let

$$(u, V_u) \ominus (u', V_{u'}) := (u - u', \{v - v' : v \in V_u, v' \in V_{u'}, v' \sqsubseteq v\}).$$

Feasibility at Building-Block Level

Define the auxiliary cost function c' by

$$(c')^{(i)} := \begin{cases} 0 & \text{if } z_1^{(i)} \geq 0 \\ -1 & \text{if } z_1^{(i)} < 0 \end{cases}, \text{ for } i = 1, \ldots, m + Nn.$$

Consider the two-stage program

$$\min\{35x_1 + 40x_2 + \frac{1}{N}\sum_{\nu=1}^{N} 16y_1^\nu + 19y_2^\nu + 47y_3^\nu + 54y_4^\nu :$$

$$\begin{aligned} x_1 + y_1^\nu + y_3^\nu &\geq \xi_1^\nu, \\ x_2 + y_2^\nu + y_4^\nu &\geq \xi_2^\nu, \\ 2y_1^\nu + y_2^\nu &\leq \xi_3^\nu, \\ y_1^\nu + 2y_2^\nu &\leq \xi_4^\nu, \end{aligned}$$

$$x_1, x_2, y_1^\nu, y_2^\nu, y_3^\nu, y_4^\nu \in \mathbb{Z}_+\}$$

Here, the random vector $\xi \in \mathbb{R}^s$ is given by the scenarios $\xi^1, \ldots, \xi^N$, all with equal probability $1/N$. The realizations of (ξ_1^ν, ξ_2^ν) and (ξ_3^ν, ξ_4^ν) are given by uniform grids (of differing granularity) in the squares $[300, 500] \times [300, 500]$ and $[0, 2000] \times [0, 2000]$, respectively. Timings are given in CPU seconds on a SUN Enterprise 450, 300 MHz Ultra-SPARC.
It took 3.3 seconds to compute $\mathcal{H}_\infty$ altogether consisting of 1438 building blocks arranged into 25 pairs (u, V_u). Aug($\mathcal{H}_\infty$) then gives the times needed to augment the solution $x_1 = x_2 = y_1^\nu = y_2^\nu = 0$, $y_3^\nu = \xi_1^\nu$, and $y_4^\nu = \xi_2^\nu$, $\nu = 1, \ldots N$ to optimality.

Example	(ξ_1, ξ_2)-grid	(ξ_3, ξ_4)-grid	scen.	var.	Aug($\mathcal{H}_\infty$)	CPLEX	dualdec
1	5×5	3×3	225	902	1.52	0.63	> 1800
2	5×5	21×21	11025	44102	66.37	696.10	–
3	9×9	21×21	35721	142886	180.63	$>$ 1 day	–

Although further exploration is necessary, the above table seems to indicate linear dependence of the computing time on the number N of scenarios, once $\mathcal{H}_\infty$ has been computed.

II

Unit Commitment – A Recurring Issue in Power Management

Step Back in Time for 101 years

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Zschornewitz

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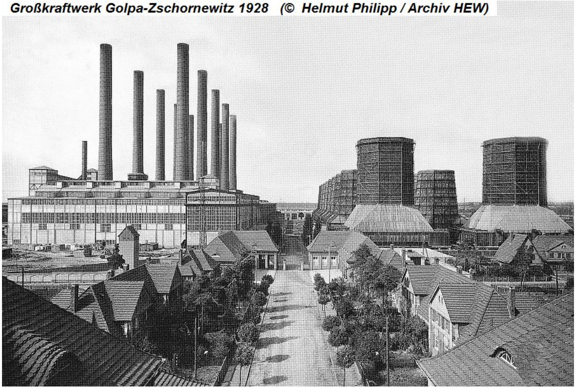

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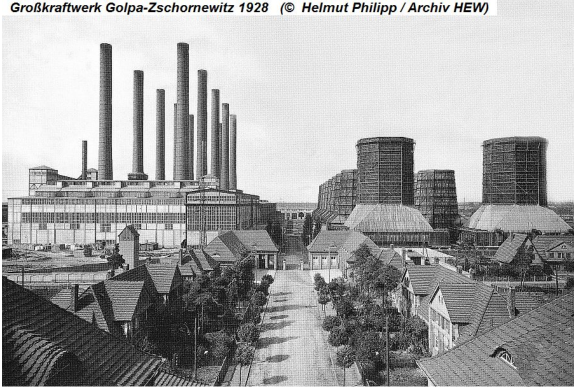

- Build within one year (Groundbreaking March 24, 1915, First Turbine (16 MW) in Operation December 15, 1915, 1916: 8×16 MW installed)

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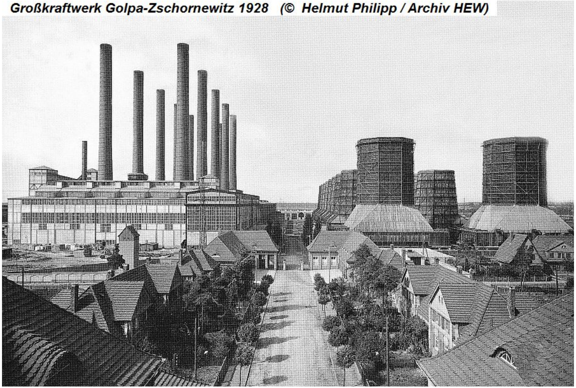

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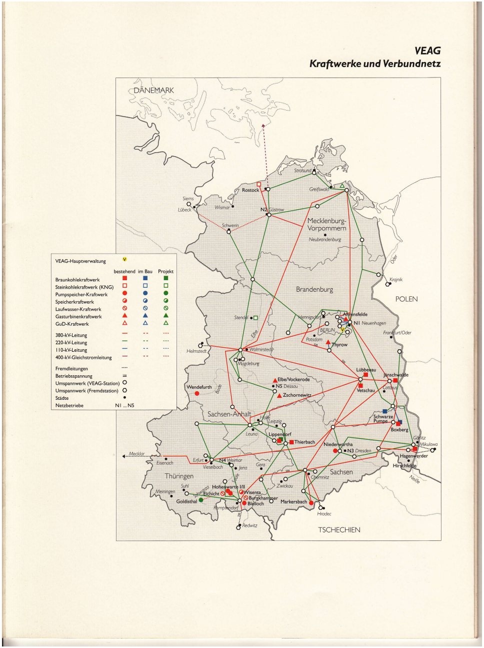

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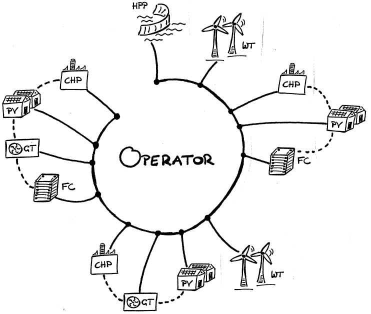
2006 Virtual Power Plant

1985 VEAG in (East Germany)



CHP : Combined heat and power units
FC : Fuel cells
GT : Gas turbines
HPP : Hydro power plants
PV : Photovoltaics
WT : Wind turbines

Specification (Mixed-Integer Linear Program – When Deterministic)

Unit Commitment for a hydro-thermal system (early VEAG + Vattenfall)

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Variables:

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Constraints:

- connecting units: load balances, reserve balances, ramping
- for individual units: output bounds, minimum up- and down-times, water management in psp,

Unit Commitment Under UNCERTAINTY

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Unit Commitment under Uncertainty Fourty Years Ago

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$$Q_{\mathbb{E}}(\xi_1) := \int_\Omega \left[c_1^\top \xi_1 + \min_{\xi_2 \in X_2} \left\{ c_2^\top \xi_2 \; : \; A_2 \xi_2 = b(\omega) - A_1 \xi_1 \right\} \right] \mathbb{P}(d\omega)$$

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Objective:

$f(\xi_1, .)$ random cost profile for operation and switching of thermal units inuced by start-up/shut-down scheme ξ_1

$$Q_{\mathbb{E}}(\xi_1) := \int_\Omega f(\xi_1,\omega)\,\mathbb{P}(d\omega) \text{ --- Expected Value -- Risk Neutral Model}$$

Unit Commitment under Uncertainty over the Years

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$$\min_x \left\{ \underbrace{c^\top x + \min_y \left\{ q^\top y \,:\, Wy = h(\omega) - Tx,\, y \in Y \right\}}_{f(x,\omega)} \,:\, x \in X \right\}$$

- 1985: Load the only quantity with relevant uncertainty - Risk neutral models, only !

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- 2010: Congestion and capacity management under uncertain in- and outputs

  $f(x, z(\omega))$ –
  x pre-commitment so that renewables' inflow z compensated with
  minimal re-commitment/re-dispatch and without overloading grid components

III

Some Thoughts on Suitable Mathematics

Viewpoints

(I) Ill-posed optimization problem
Destructive – remove stochasticity swiftly,

$$\min \{f(x,\omega) \; : \; x \in X\}$$

As long as ω is unknown, it makes no sense to address optimality.

Remedy:

Arrive at a deterministic problem by "removing ω in formal manner".

- Replace ω by its expectation $\mathbb{E}[\omega]$ and solve $\min \{f(x, \mathbb{E}[\omega]) \; : \; x \in X\}$
- Consider expected value $\mathbb{E}[f(x,\omega)]$ and solve $\min \{\mathbb{E}[f(x,\omega)] \; : \; x \in X\}$
- Apply a statistical parameter $\mathcal{S}$ and solve $\min \{\mathcal{S}[f(x,\omega)] \; : \; x \in X\}$

(II) Optimizing or ranking in a family of random variables

Constructive: Be happy about havng stochastic information on the uncertain problem ingredients. Make active use of it.

$$\{f(x,.) : \Omega \to \mathbb{R}\}_{x \in X}$$

Remedy:

Arrive at a deterministic problem by implementing your attitude towards risk .

- Risk neutral: Apply expectation $\mathbb{E}$ to $f(x,\omega)$ and solve

$$\min \{\mathbb{E}[f(x,\omega)] \ : \ x \in X\}$$

- Risk averse by criterion: Apply some risk measure $\mathcal{R}$ and solve

$$\min \{\mathcal{R}[f(x,\omega)] \ : \ x \in X\}$$

- Risk averse by constraint: Rank according to some stochastic order. Introduce a benchmark random variable $b(\omega)$ leading to the constraint

$$\{x \in X \ : \ f(x,\omega) \preceq b(\omega)\}$$

Solution by Scenario Decomposition

$$Q_{\mathbb{E}}(\xi_1) := \int_{\Omega} \left[c_1^\top \xi_1 + \min_{\xi_2 \in X_2} \left\{ c_2^\top \xi_2 \ : \ A_2 \xi_2(\omega) = b(\omega) - A_1 \xi_1 \right\} \right] \mathbb{P}(d\omega)$$

Assume the rhs $b(\omega)$ is the only random ingredient, and let it follow a finite discrete probability distribution

with scenarios $b_1, \ldots, b_\omega, \ldots, b_S$ and probabilities $\pi_1, \ldots, \pi_\omega, \ldots, \pi_S$

Then $\min\{Q_{\mathbb{E}}(\xi_1) \ : \ \xi_1 \in X_1\}$ is equivalent to the following large-scale block angular mixed-integer linear program

$$\min \left\{ c_1^\top \xi_1 + \sum_{\omega=1}^{S} \pi_\omega c_2^\top \xi_{2\omega} \ : \ \begin{array}{lllll} A_1\xi_1 + A_2\xi_{21} & & & = & b_1 \\ \vdots & \ddots & & & \vdots \\ A_1\xi_1 & + A_2\xi_{2\omega} & & = & b_\omega \\ \vdots & & \ddots & & \vdots \\ A_1\xi_1 & & + A_S\xi_{2S} & = & b_S \end{array} \right.$$
$$\left. \xi_1 \in X_1,\ \xi_{2\omega} \in X_2,\ \omega = 1, \ldots, S \right\}$$

Scenario Decomposition

Basic Idea: Lagrangean Relaxation of Nonanticipativity[Carøe/Sch.1999]:

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Introduce copies $\xi_{11}, \ldots, \xi_{1\omega}, \ldots, \xi_{1S}$ of ξ_1 and add $\xi_{11} = \ldots = \xi_{1\omega} = \ldots = \xi_{1S}$.

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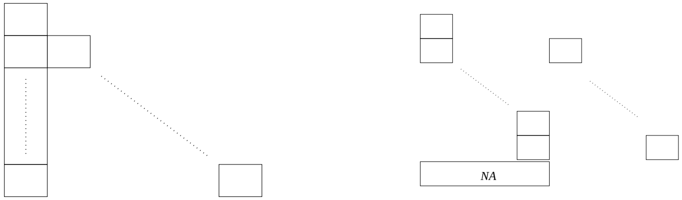

This includes solving the Lagrangean Dual which is a non-differentiable convex optimization problem.

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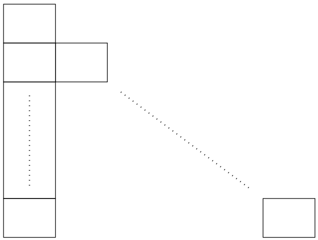
This works nicely as long as mixed-integer linear programming formulation has the block structure



III

Congestion Management in Power Nets

Load Flow Models – AC, DC, Ohmic Losses

Graph $G = (V, E)$ (undirected)

with nodes $v \in V = \{1, \ldots, n\}$, edges $e \in E \subseteq V \times V$.

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For all edges in E (AC) Load Flow Equations

$$
\begin{aligned}
p_{vl} &= U_v^2 g_{vl} - U_v U_l g_{vl} \cos \theta_{vl} - U_v U_l b_{vl} \sin \theta_{vl} && \forall vl \in E \\
q_{vl} &= U_v U_l b_{vl} \cos \theta_{vl} - U_v U_l g_{vl} \sin \theta_{vl} - U_v^2 (b_{vl} + b_{vl}^0) && \forall vl \in E
\end{aligned}
$$

Load Flow Models

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From AC equations, only the first remains and becomes:

DC Load Flow Equation

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Load Flow Models

DC Load Flow with Ohmic Losses

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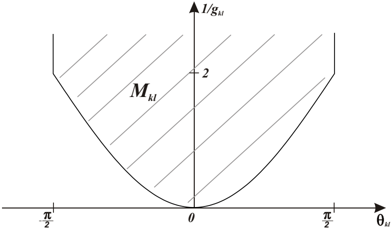
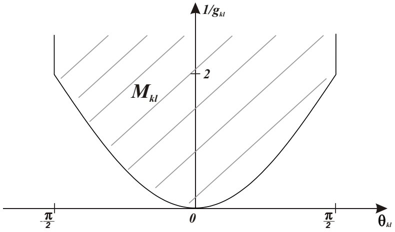


Loss on $vl \in E$

$$\nu_{vl} = g_{vl}(U_v^2 + U_l^2) - 2 g_{vl} U_v U_l \cos(\theta_v - \theta_l) \qquad \forall vl \in E$$

Load Flow Models

DC Load Flow with Ohmic Losses



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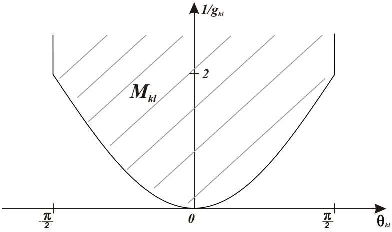
DC Load Flow with Ohmic Losses



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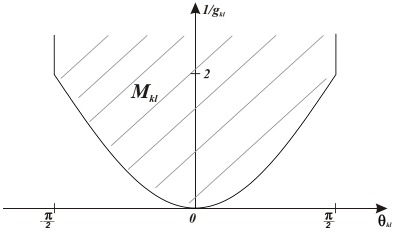

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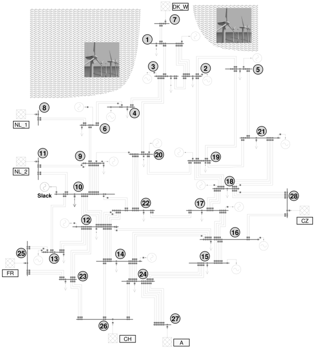

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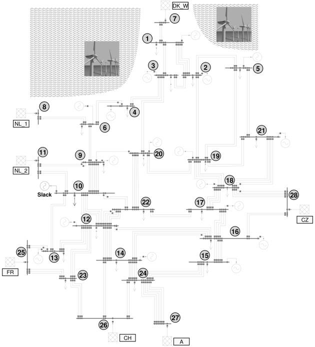

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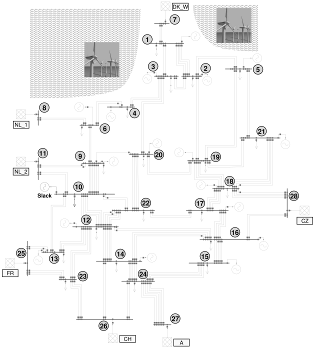

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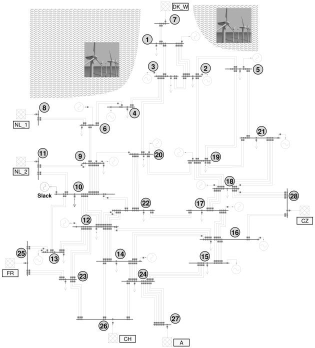

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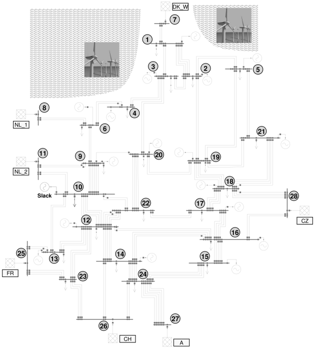

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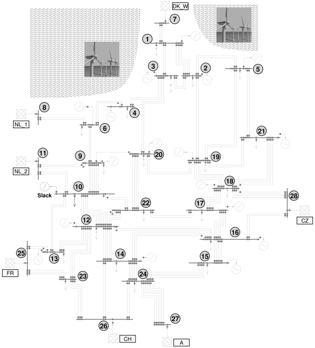
Daniel Waniek (Energy Science, PhD 2011)



- Optimal pre-commitment/ pre-dispatch to avoid grid congestion with re-dispatch/re-commitment.
- DC model with Ohmic losses and polyhedral approximation of convex nonlinearities. incl. code.
- Mixed-integer linear models with switching.
- Stochastics, decomposition.
- Variation of wind infeed rate from 40 via 80 to 100%.

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  - Double checking flows with commercial solver `NEPLAN` with switching decisions and fixed by our code.
- Evaluation of losses over-estimation caused by relaxation.

Congestion Management under Inflow of Renewables - Wind

Numerical Tests

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		Ref. (AC)	Opt. (DoDu)	Opt. (AC)	Ref. (AC)	Opt. (DoDu)	Opt. (AC)
Wind	[-]		40 %			80 %	
Generation Cost	[T€]	1231	1200	1201	971	986	987
Import	[MW]	5347	5882	5882	5347	5483	5483
Export	[MW]	3472	3125	3125	3472	3125	3125
Grid Losses	[MW]	444	424	434	1016	700	709
Overload of grid components	[-]	no	no	no	yes	no	no

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		Ref. (AC)	Opt. (DoDu)	Opt. (AC)
Wind	[-]		100 %	
Generation Cost	[T€]	858	945	945
Import	[MW]	5437	5483	5483
Export	[MW]	3472	3125	3125
Grid Losses	[MW]	1468	762	768
Overload of grid components	[-]	yes	no	no