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RICAM Special Semester on Optimization

Workshop 1
New trends in PDE constrained optimization

Book of Abstracts

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Additive manufacturing constraints in topology optimization of structures

Grégoire Allaire
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Abstract

Additive manufacturing (or 3-d printing) is a new exciting way of building structures without any restriction on their topologies. Unfortunately, it comes with its own restrictions as the presence of undesired thermal residual stresses and the difficulty to build overhang regions, which sometimes require so-called additional support structures.

Therefore, we propose some new functionals, to be used as constraints in shape and topology optimization problems, as a means to enforce the manufacturability of structures by additive manufacturing processes. Instead of considering merely the final shape, they aggregate objective functions (like compliance) for all the intermediate structures of the shape appearing in the course of its layer by layer assembly and subject to their self-weights or to thermal loadings due to the fabrication process. Typically, these functionals penalize overhangs, thermal residual stresses and thermal deformations in the build direction. We compute their shape derivatives and implement them into a shape and topology optimization algorithm based on the level set method. We discuss the merits of these various models and present some numerical validations on concrete examples. This is a joint work with many colleagues, including B. Bogosel, C. Dapogny, L. Jakabcin.

Multiobjective Parameter Optimization of Elliptic PDEs using the Reduced Basis Method

Stefan Banholzer
University of Konstanz

Abstract

This talk presents joint work with Bennet Gebken, Sebastian Peitz, Michael Dellnitz (all Paderborn University) and Stefan Volkwein (University of Konstanz).

Many optimization problems in applications can be formulated using several objective functions, which are conflicting with each other. This leads to the notion of multiobjective or multicriterial optimization problems.
In this talk we present a homotopy method for solving general multiobjective optimization problems, which also exploits the hierarchical structure of the solution set. This is then applied to the multiobjective parameter optimization of an elliptic convection-diffusion-reaction equation with the cost functions either being of tracking type or measuring the parameter costs.
In the course of the homotopy method, numerous scalar KKT-systems have to be solved, which involves the repeated solution of the elliptic PDE and its adjoint equation. Thus, the Reduced Basis (RB) method is introduced as an approach for model-order reduction. By modifying the KKT-systems according to the desired exactness of the reduced-order model, a covering of the original solution set can be computed. Theoretical convergence results as well as practical examples are presented to validate our numerical algorithm.

Observability inequalities for discrete parabolic equations and systems

Franck Boyer
Institut de Mathématiques de Toulouse

Abstract

It is well known that semi discrete or fully discrete parabolic equations or systems are usually not uniformly observable with respect to the discretisation parameters even though the associated continuous PDE is observable. We will discuss in this talk what should be a suitable relaxation of those observability inequalities in the discrete setting, in connection with the penalised HUM approach. We will give a short review of various results of the literature in this direction and their application to the numerical approximation of null-controls for such PDEs.

Optimal Shape Design for the p-Laplace Eigenvalue Problem
Farid Bozorgnia
Tecnico Lisboa

Abstract

In this talk, we consider a shape optimization problem corresponding to the p-Laplace operator. Given a density function in a rearrangement class generated by a step function, we aim to find the density such that the principal eigenvalue is as small as possible. We obtain some qualitative aspects of the optimizer and then we determine nearly optimal sets which are approximations of the minimizer for specific ranges of parameters values.

Next, a numerical algorithm to derive the optimal shape is given and we show that the numerical procedure converges to a local minimizer. Numerical illustrations are provided for different domains to show the efficiency and practical suitability of our approach.

Nonlinear infinite-horizon control using generalized Lyapunov equations

Tobias Breiten
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Abstract

It is well-known that the solution to the infinite-horizon linear-qudratic control problem is characterized by the algebraic Riccati equation. For nonlinear dynamics, one instead has to focus on the Hamilton-Jacobi-Bellman equation, a nonlinear PDE that suffers from the curse of dimensionality. This talk discusses approximation techniques for the HJB equation based on a series of generalized Lyapunov equations. Theoretical and numerical results as well as future challenges for these types of equations are presented.

This is joint work with Karl Kunisch and Laurent Pfeiffer.

On the relations between principal eigenvalue and torsional rigidity

Giuseppe Buttazzo
Department of Mathematics – University of Pisa

Abstract

The relations between principal eigenvalue of the Laplace operator and torsional rigidity are studied in the class of general domains, convex domains, and domains with a small thickness. This is of help to provide some bounds for the Blasche-Santaló diagram of the two quantities.

This is a joint work with M. van den Berg and A. Pratelli.

A sequential quadratic Hamiltonian scheme for solving optimal control problems with non-smooth cost functionals

Alfio Borzì
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Abstract

A sequential quadratic Hamiltonian (SQH) method for solving different classes of non-smooth and non-convex ODE and PDE optimal control problems is presented. The solutions to these problems are characterised by the Pontryagin's maximum principle (PMP), which is also the starting point for the formulation of the SQH scheme.

Convergence of the SQH method is discussed for different benchmark control problems. These problems include linear and nonlinear differential models with linear and bilinear control mechanisms, non-convex and discontinuous costs of the controls, and the case of state constraints. For each problem, a theoretical discussion of the PMP optimality condition is given and results of numerical experiments are presented that demonstrate the large range of applicability of the SQH scheme.

This talk provides a partial review of the work presented in the following papers.

- Tim Breitenbach and Alfio Borzì,
  A sequential quadratic Hamiltonian method for solving parabolic optimal control problems with discontinuous cost functionals,
  Journal of Dynamical and Control Systems, 25 (2019), 403–435.
- Tim Breitenbach and Alfio Borzì,
  On the SQH scheme to solve non-smooth PDE optimal control problems",
  Journal of Numerical Functional Analysis and Optimization, 40 (2019), 1489–1531.
- Tim Breitenbach and Alfio Borzì, A sequential quadratic Hamiltonian scheme for solving non-smooth quantum control problems with sparsity", submitted to JCAM, 2019.
- Tim Breitenbach and Alfio Borzì, The Pontryagin maximum principle for solving Fokker-Planck optimal control problems", submitted to COAP, 2019.

This is joint work with Tim Breitenbach (University of Würzburg).

Analysis of control problems of nonmonotone semilinear elliptic equations

Eduardo Casas
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Abstract

In this talk we consider optimal control problems governed by a semilinear elliptic equation. The equation is nonmonotone due to the presence of a convection term, despite the monotonocity of the nonlinear term. The resulting operator is neither monotone nor coervive. However, by using conveniently a comparison principle we prove existence and uniqueness of solution for the state equation. In addition, we prove some regularity of the solution and differentiability of the relation control-to-state. This allows us to derive first and second order conditions for local optimality.

Optimal control of nonsmooth equations

Christian Clason
University of Duisburg-Essen

Abstract

This talk is concerned with PDE-constrained optimization problems where the PDE constraint involves Lipschitz continuous but not classically differentiable terms. Correspondingly, the control-to-state mapping is not differentiable either, and classical approaches fail. In particular, there exists a zoo of optimality conditions of different strengths, roughly corresponding to different generalized derivatives of the control-to-state mapping. We derive such optimality conditions for model problems and discuss how they can be used for their numerical solution.

This talk is based on joint work with Constantin Christof, Christian Meyer, Vu Huu Nhu, and Arnd Rösch.

Mosquito population control strategies for fighting against arboviruses

Michel Duprez
University of Paris-Dauphine

Abstract

In the fight against vector-borne arboviruses, an important strategy of control of epidemic consists in controlling the population of the vector, *Aedes* mosquitoes in this case. Among possible actions, two techniques consist either in releasing sterile mosquitoes to reduce the size of the population (Sterile Insect Technique) or in replacing the wild population by one carrying a bacteria, called *Wolbachia*, blocking the transmission of viruses from insects to humans. This talk is devoted to studying the issue of optimizing the dissemination protocol for each of these strategies, in order to get as close as possible to these objectives. Starting from a mathematical model describing population dynamics, we will study the control problem and introduce the cost function standing for *population replacement* and *sterile insect technique*. Then, we will establish some properties of the optimal control and illustrate them with numerical simulations.

Null space gradient flows for shape optimization of multiphysics systems
Florian Feppon
CMAP (École polytechnique) – Safran Tech

Abstract

In this workshop, I will present my PhD research work focusing on shape optimization of coupled thermal fluid-structure systems with Hadamard's method of boundary variations. I will detail how several classical cornerstones of shape optimization can be addressed in order to tackle long term industrial needs and requirements: (i) the derivation of a single, unified formula for the Fréchet derivative of any arbitrary shape functional (ii) the numerical discretization of shapes and their updates throughout the optimization, namely with a level-set based mesh evolution method (iii) the choice of an efficient and reliable optimization algorithm that allows to handle an arbitrary number of equality and inequality constraints

For the latter part, we rely on a generalization of the celebrated gradient flow to optimization problems featuring equality and inequality constraints. More classical optimization methods are often difficult to use because of the infinite dimensional context or the need for tuning finely algorithm parameters. Inequality constraints are specifically addressed by solving a dual quadratic programming subproblem which allows to detect the subset of these to which the optimization trajectory needs to remain tangent.

A variety of 2D and 3D shape optimization test cases will be presented to illustrate the numerical efficiency and robustness of our algorithm.

Optimal Experimental Design for Large-Scale Bayesian Inverse Problems via Multi-PDE-Constrained Optimization

Omar Ghattas
Oden Institute, The University of Texas at Austin, USA

Abstract

Bayesian inference provides a systematic and principled framework for learning complex physical models from complex observational data under uncertainty. In the resulting Bayesian inverse problem, we employ observational data and a model mapping parameters to observables to infer unknown parameters and quantify their uncertainty, in the form of a posterior probability distribution. But this begs the question: how do we acquire the "best" data? (i.e., sensor locations, what is measured and when, etc.) This is the optimal experimental design (OED) problem to determine the optimal design of the data acquisition system.

Here we address the specific OED objective of maximizing the expected information gain (EIG), i.e., the expectation (over the data) of the Kullback—Leibler (KL) divergence from posterior to prior. Naive evaluation of EIG is intractable for large-scale problems due to the large number of samples (and thus forward PDE solves) required by double loop Monte Carlo sampling of the KL divergence and its expectation.

To overcome the prohibitive nature of double loop Monte Carlo, we invoke an approximation of the EIG based on the Laplace approximation of the posterior, which is a Gaussian distribution centered at the point that maximizes the posterior probability (the MAP point) with covariance operator given by the inverse of the Hessian at that point. This permits the KL divergence to be expressed in terms of the log-determinant and trace of the preconditioned Hessian of the log posterior. Rapid spectral decay and a randomized eigensolver allow us to estimate these invariants at a cost—measured in number of forward PDE solves—that is independent of the number of uncertain parameters and design variables. This results in an optimization problem to find the optimal experimental design that includes not only the PDEs governing the inverse problem as constraints, but also the adjoint PDEs and the gradient equation (to define the MAP point), as well as second order forward and adjoint PDEs and an eigenvalue problem for the Hessian operator (to define the trace and determinant). Despite the formidable appearance of this multi-PDE constrained optimization problem, we show that it can be solved at a cost, measured in PDE solves, that is independent of the dimensions of both the (discretized) random parameter field and the observational data, and only weakly dependent on the design dimension.

This work is joint with Umberto Villa (Washington University, St. Louis, USA)

The reduced basis method in constrained optimal control with PDEs
Michael Hinze
Mathematisches Institut, Universität Koblenz-Landau

Abstract

With this talk we present a novel reduced-basis approach for optimal control problems with constraints, which seems to deliver lower dimensional RB spaces as reported in the literature so far for the same problem class, but with the same approximation properties, and which allows to prove an error equivalence as known from finite element a posteriori error analysis. We also present convergence analysis for the method. Numerical test confirm our theoretical findings.

This is joint work with Ahmad Ahmad Ali.

Variational discretization of PDE constrained optimal control problems with measure controls

Evelyn Herberg
Mathematisches Institut, Fachbereich 3: Mathematik/Naturwissenschaften, Universität Koblenz-Landau, Universitätsstraße 1, 56070 Koblenz

Abstract

joint work with Michael Hinze and Henrik Schumacher

We consider variational discretization of a parabolic optimal control problem governed by space-time measure controls. Via the discretization of the test space and the optimality conditions, an implicit discretization of the control is achieved. We utilize this property and choose the Petrov-Galerkin-ansatz and -test space that will induce maximal discrete sparsity, i.e. Dirac-measures in space-time. Furthermore, we compare our approach to a full discretization of the problem. Numerical experiments highlight the features of our discrete approach.

Optimal Control of Two-phase Flow

Christian Kahle
Universität Koblenz–Landau

Abstract

We present results on optimal control of time discrete two-phase flows. The fluid is modeled by a thermodynamically consistent diffuse interface model that is proposed in [H. Abels, H. Garcke, G. Grün, Thermodynamically consistent, frame indifferent diffuse interface models for incompressible two-phase flows with different densities, M3AN, 22(3), 2012] and allows for fluids of different densities and viscosities.

In earlier work [H. Garcke, M. Hinze, C. Kahle, A stable and linear time discretization for a thermodynamically consistent model for two-phase incompressible flow, APNUM, 99, 2016] we proposed an energy stable time discretization for this model that mimics the energy behaviour of the continuous model. We now employ this scheme to derive existence of optimal controls for a time discrete optimal control problem and also first order neccessary conditions.

The control aim is to obtain a desired distribution of the two phases in the system. For this we investigate three control actions. We use tangential Dirichlet boundary control and distributed control with controls acting inside the domain. We further consider the inverse problem of finding an initial distribution such that the evolution over a given time horizon starting from this value is close to a desired distribution.

This is cooperation with Harald Garcke (Universität Regensburg) and Michael Hinze (Universität Koblenz–Landau)

Optimal control of PDEs with point-wise state constraint

Kazufumi Ito
Department of Mathematics, North Carolina State University

Abstract

In this talk we discuss a point-wise state constraint problem for a general class of optimal control problems. We use the penalty formulation and derive the necessary optimality condition based on the Lagrange multiplier theory.The existence of Lagrange multiplier associated with the point-wise state constraint as a measure is established. Also we develop a semi-smooth Newton method for the penalty formulation. Numerical tests are presented for parabolic, elliptic and wave equation control problems. The results show that the state constraint optimal control method enables us to develop a much powerful and useful control law. For example one can use to prevent blow-up solutions.

Optimal control of a semilinear heat equation with bilinear control-state terms subject to state and control constraints

Axel Kröner
Humboldt Universität Berlin

Abstract

In this paper we consider an optimal control problem governed by a semilinear heat equation with bilinear control-state terms and subject to control and state constraints. The state constraints are of integral type, the integral being with respect to the space variable. The control is multidimensional. The cost functional is of a tracking type and contains a linear term in the control variables. We derive second order necessary and sufficient conditions relying on the Goh transform, the concept of alternative costates, and quasi-radial critical directions.

This is joint work with Frédéric Bonnans and Soledad Aronna.

Concepts for Breaking the Curse of Dimensionality for the Optimal Control HJB Equation

Karl Kunisch

University of Graz, Austria, and Radon Institute, Austrian Academy of Sciences

Abstract

Expressing optimal controls in closed loop form naturally leads to Hamilton Jacobi Bellman equations if the control system is not of linear-quadratic type. The analysis of these equations in the context of viscosity solutions is fairly well understood by now. Due to the curse of dimensionality their numerical realization, however, remains to be a significant challenge. In this lecture I discuss the underlying concepts of two approaches which we follow to partially alleviate these difficulties. Both of them are geared to eventually solve HJB equations for optimal stabilisation of models described by PDEs.

The first of the proposed methods combines a tensor calculus based ansatz for the value function with a Newton-like iterative method for the solution of the resulting nonlinear HJB-system.

The second one is based on a learning approach for the approximation of the feedback gains by neural networks.

Numerical examples demonstrate that there can be a significant difference between the optimal trajectories based on solving the HJB equation and applying a Riccati based approach to the linear-quadratic approximation of the original system, both in the transient, and the asymptotic phases.

The first part of the lecture is based on joint work with S. Dolgov, University of Bath and D. Kalise, Imperial College, the second part with D. Walter, RICAM Linz.

Turnpike in optimal shape design

Gontran Lance
Sorbonne Université, Université Paris-Diderot SPC, CNRS, Inria, Laboratoire Jacques-Louis Lions, équipe CAGE, F-75005 Paris

Abstract

We investigate the turnpike problem in optimal control, in the context of time-evolving shapes. We focus here on the heat equation model where the shape acts as a source term, and we search the optimal time-varying shape, minimizing a final or integral quadratic criterion. We establish in the first case existence of optimal solutions and we prove that optimal time shape and optimal static shape verify the exponential turnpike property for the Hausdorff distance. In the second case, we establish existence of optimal solutions under some appropriate sufficient conditions. We provide necessary conditions for optimality in terms of usual adjoint equations and then, thanks to strict dissipativity properties, we prove that state and adjoint satisfy a measure-turnpike property, meaning that the extremal time-varying solution remains essentially close to an optimal solution of an associated static problem. We illustrate the turnpike phenomenon in shape design with several numerical simulations.

Numerical Analysis of Sparse Initial Data Identification for Parabolic Problems

Dmitriy Leykekhman
University of Connecticut

Abstract

In this talk we consider a problem of initial data identification from the final time observation for homogeneous parabolic problems. It is well-known that such problems are exponentially ill-posed, has strong smoothing property and very challenging computationally. We are interested in a situation when the initial data we intend to recover is known to be sparse , i.e. has Lebesgue measure zero. We formulate the problem as an optimal control problem and incorporate the sparsity information on data into the structure of the objective functional. In particular in the cost functional contains penalization term of the control in a measure space, which makes the analysis of the problem rather technical. To approximate the problem we use fully discrete continuous piecewise linear finite elements in space and discontinuous Galerkin finite elements of arbitrary degree in time and obtain convergence rates for the state error. In the case when the initial data consists of a linear combination of Dirac measures, we obtain first order convergence rates in space and optimal superconvergent rates in time. The key tool for the analysis are the sharp smoothing type pointwise finite element error analysis for homogeneous parabolic problems. We also illustrate the problem with several numerical experiments.

Fast algorithms for nonlinear optimal control for diffeomorphic registration

Andreas Mang
Department of Mathematics, University of Houston, TX, US

Abstract

We present efficient algorithms for nonlinear optimal control problems for diffeomorphic registration. Our contributions are the design of effective numerical methods and fast computational kernels, and their analysis.

In general, the problem of diffeomorphic registration is follows: Given two views of the same scene acquired from different fields of vision or at different points in time, we seek a spatial transformation y that relates points in one view of an object to its corresponding points in another view of the same object. The inputs to our problem are, in general, two images (2D pictures or 3D volumes). We restrict the space of admissible maps $\mathcal{Y}$ to the group of diffeomorphisms. We use an optimal control formulation and invert for a time-dependent velocity field v that parameterizes the sought-after spatial transformation y. In its simplest form, the PDE constraints of our formulation are given by a hyperbolic transport equation for the image intensities. Suitable regularity requirements for v ensure that the map y exists, and is a diffeomorphism, i.e., y is a bijection and has a smooth inverse. We have extended our basic formulation by constraints on the divergence of the velocity to restrict y to near-incompressible diffeomorphisms. In addition to that, we will discuss an initial value control problem; the PDE constraints are a transport equation for the image intensities and the EPDiff equation that models the evolution of critical paths on the manifold of diffeomorphisms.

Our solvers are founded on state-of-the-art techniques in scientific computing to enable fast convergence and short execution times. Our software framework is termed CLAIRE, and has been deployed in high-performance computing platforms. More recently, we have introduced dedicated GPU kernels to CLAIRE. This, in combination with efficient numerical algorithms enables the solution of the associated control problems to high accuracy in well under 10 seconds for clinically relevant problems in three dimensions on a single GPU. We will discuss different building blocks of our solver, showcase results for real and synthetic data, and study the rate of convergence, time-to-solution, numerical accuracy, and scalability of our solvers for different formulations and implementations.

This is joint work with George Biros, Malte Brunn, Naveen Himthani, Felix Huber, and Miriam Mehl.

Optimal Control of Perfect Plasticty

Christian Meyer
TU Dortmund, Faculty of Mathematics

Abstract

The talk is concerned with optimal control of quasi-static perfect elasto-plasticity at small strains. Perfect plasticity is characterized by a lack of coercivity (due to the absence of any hardening) and therefore provides particular challenges. To be more precise, the associated displacement field is in general not unique and may be rather irregular (space of bounded deformation), while the stress field is a unique L^2-function. For this reason, we consider two different optimal control problems, stress tracking and displacement tracking. Both optimal control problems are non-smooth due to the maximal monotone map associated with the evolution law for the plastic strain. We therefore apply a Yosida-type regularization of this mapping in order to obtain smooth optimal control problems. In case of the stress tracking, the uniqueness and regularity of the stress field allows us to prove convergence of optimal solutions of the regularized problems to minimizers of the original non-smooth problem. In contrast to this, the situation concerning the displacement tracking is more difficult. In this case, we need an additional (rather non-physical) control variable and additional smoothness properties of at least one optimal displacement field for similar convergence results. The regularized problems are classical smooth optimal control problems, that can be treated by standard methods. Our regularization approach can also be used for numerical purposes, as we demonstrate by means of a two-dimensional stress tracking example.

This is joint work with Stephan Walther (TU Dortmund).

On the use of the damped Newton method to solve direct and controllability problems for parabolic PDEs

Arnaud Münch

Laboratoire de Mathématiques Blaise Pascal, Université Clermont Auvergne, Clermont-Ferrand, France

Abstract

We introduce and analyze space-time least-squares methods to approximate direct and controllability problems associated to nonlinear PDEs.

In a first part, we focus on the incompressible Navier-Stokes system in two and three space dimension. Using an appropriate descent direction, we construct a globally convergent sequence (strongly in $L^2(0,T;\boldsymbol{V})) \cap H^1(0,T;\boldsymbol{V}'))$ to a solution of the NS system. Except for the first iterates (in finite number related to the viscosity constant), the convergence is quadratic.

In a second part, we extend the method to the controllability of sub-linear (uniformly controllable) scalar heat equations and construct a globally convergent sequence in $L^2(q_T)$ to a null control.

In both cases, it turns out that such (particular) convergent sequences (minimizing for the least-squares functional) are related to those obtained when the damped Newton method is considered (to address underlying weak formulations).

Numerical experiments in two space dimension illustrate the theoretical results. Joint works with Jérome Lemoine (Clermont-Ferrand, France) and Irene Marín-Gayte (Seville, Spain).

Fast Interior Point Solvers and Preconditioning for PDE-Constrained Optimization

John Pearson
University of Edinburgh

Abstract

In this talk we consider the effective numerical solution of PDE-constrained optimization problems with additional box constraints on the state and control variables. Upon discretization, these may give rise to problems of quadratic or nonlinear programming form: a sensible solution strategy is to apply an interior point method, provided one can solve the large and structured matrix systems that arise at each Newton step. We therefore consider fast and robust preconditioned iterative methods for these systems, examining two cases: (i) where L^2 norms measure the misfit between state and desired state, as well as the control; (ii) with an additional L^1 norm term promoting sparsity in the control. Having motivated and derived our recommended preconditioners, and shown some theoretical results on saddle point systems, we present numerical results demonstrating the potency of our solvers.

This talk is based on work with Jacek Gondzio, Margherita Porcelli, and Martin Stoll.

Optimal location of resources for biased movement of species

Yannick Privat
IRMA, Univ. Strasbourg, France

Abstract

In this work, we are interested in the analysis of optimal resources configurations (typically foodstuff) necessary for a species to survive. For that purpose, we use a logistic equation to model the evolution of population density involving a term standing for the heterogeneous spreading (in space) of resources. The principal issue investigated in this talk writes: How to spread in an optimal way resources in a closed habitat? This problem can be recast as the one of minimizing the principal eigenvalue of an elliptic operator involving a drift term with respect to the domain occupied by resources, under a volume constraint. We investigate both the issues of: 1) optimal configurations existence, 2) qualitative description of minimizers. This is a joint work with Idriss Mazari and Grégoire Nadin.

Generalized derivatives for the solution operator of obstacle problems

Anne-Therese Rauls
TU Darmstadt

Abstract

We discuss differentiability properties for the solution of the obstacle problem with respect to different types of controls and derive generalized derivatives for the solution operator of a general class of obstacle problems. The constructed generalized derivatives are solution operators of Dirichlet problems on the respective inactive sets. We adapt the strategy for two prototype examples involving a stochastic optimal control problem and a shape optimization problem.

Some challenges of four-dimensional data assimilation problems

Juan Carlos De los Reyes
Centro de Modelización Matemática, Escuela Politécnica Nacional, Quito, Ecuador

Abstract

We will consider variational four-dimensional data assimilation problems in function space, and discuss topics such as well-posedness, regularity of Lagrange multipliers and optimality conditions. The use of alternative regularizers in the tracking type cost will be discussed as well, in order to cope with possible discontinuities of the state. Finally, the optimal time and space location of observations is studied in function space, using a supervised learning approach.

Shape Optimization for Geometrically Inverse Problems

Stephan Schmidt
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Abstract

We focus on shape optimization for geometrically inverse problems, i.e., the reconstruction of inclusions from measured boundary data. Prototypical applications are geoelectric impedance tomography with elliptic differential equations or reconstruction tasks from characteristics of waves and hyperbolic PDEs. Crucial for a proper reconstruction is the use of appropriate regularization terms. As part of this presentation, we also consider the total variation of the normal as prior. This prior has considerable benefits over, e.g., a perimeter regularization when reconstructing non-smooth objects, albeit at the cost of making the shape optimization problem non-smooth. For the smooth situation, we also discuss the efficient use of shape Hessians for Newton-like schemes where the domain is the unknown.

This presentation is joint work with M. Herrmann, J. Vidal Núñez, R. Bergmann and R. Herzog.

Tensor approach to optimal control problems with fractional d-dimensional elliptic operator in constraints

Volker Schulz
Trtier University, Germany

Abstract

We introduce a tensor numerical method for solution of the d-dimensional optimal control problems with fractional Laplacian type operators in constraints discretized on large spacial grids. It is based on the rank-structured approximation of the matrix valued functions of the corresponding fractional elliptic operator. The functions of finite element (finite difference) Laplacian on a tensor grid are diagonalized by using the fast Fourier transform (FFT) matrix and then the low rank tensor approximation to the multi-dimensional core diagonal tensor is computed. The equation for the control function is solved by the PCG method with the rank truncation at each iteration step where the low Kronecker rank preconditioner is precomputed by using the canonical decomposition of the core tensor for the inverse of system matrix. The right-hand side, the solution, and the governing operator are maintained in the rank-structured tensor format. Numerical tests for the 2D and 3D control problems confirm the linear complexity scaling of the method in the univariate grid size.

The presentation is based on joint work with Gennadij Heidel, Britta Schmitt, Boris Khoromskij and Elena Khoromskaja.

Extreme event probability estimation by combining large deviation theory and PDE-constrained optimization, with application to tsunamis

Georg Stadler
Courant Institute of Mathematical Sciences, New York University

Abstract

Tsunami waves are caused by a sudden change of ocean depth (bathymetry) after an earthquake below the ocean floor. Since large tsunami waves are extreme events, they correspond to the tail part of a probability distribution, whose exploration would require impractically many samples of a Monte Carlo method. We propose an alternative method to estimate extreme probabilities using large deviation theory, which relates the probabilities of extreme events to the solutions of a one-parameter family of optimization problems. To model tsunami waves, we use the shallow water equations, which thus appear as PDE-constraints in this optimization problem. The optimization objective includes a term that measures how extreme the event is, and a term corresponding to the likelihood of bathymetry changes, which are modeled as a Gaussian random field. Preliminary numerical results with the 1D inviscid shallow water equation are presented. This is joint work with Shanyin Tong and Eric Vanden-Eijnden (both NYU).

Control of Parabolic Delay Equations

Fredi Tröltzsch
Technische Universität Berlin

Abstract

Semilinear parabolic delay differential equations are considered, where time delays occur in different ways, discrete or continuously. The delays and associated weights are the subject of optimization or stabilization. In the most general case, the delays are generated by regular Borel measures. An existence and uniqueness theorem for such delay equations and the differentiability of the mapping is discussed that associates the solution of the delay equation to the measure control or to a vector of time delays. Associated optimal control problems are sketched. Moreover, the issue of stabilization by some Pyragas type feedback is addressed. Several numerical examples are presented.

The talk is based on joint work with Eduardo Casas (Santander), Mariano Mateos (Gijón), and Martin Gugat (Erlangen).

Optimal control of hyperbolic conservation laws and convergent numerical schemes for sensitivities and adjoints

Stefan Ulbrich
TU Darmstadt

Abstract

We consider initial-boundary control problems for entropy solutions of hyperbolic conservation laws. The controls switch between smooth functions at certain switching points and the smooth parts as well as the switching points are considered as controls. Our results provide a basis to handle controlled networks of conservation laws with appropriate node conditions.

Based on a sensitivity and adjoint calculus that is also applicable for discontinuous solutions involving shocks, we obtain an adjoint-based derivative representation of the cost functional. The correct adjoint state is the so-called reversible solution of the adjoint equation. Moreover, we derive optimality conditions, where we consider in particular also the case of state constraints.

The discretization of such optimal control problems is involved, since discretize-then-optimize as well as optimize-then-discreitize approachs may converge to a wrong adjoint state. Based on a finite difference scheme in conservation form and its corresponding adjoint scheme we discuss two approaches to ensure convergence of the discrete adjoint to the correct adjoint state. We illustrate this by numerical examples.

Reduced-Order Approaches for PDE-Constrained Multiobjective Optimization

Stefan Volkwein
Department of Mathematics and Statistics, University of Konstanz, Germany

Abstract

There is hardly ever a situation where only one goal is of interest at a time. When carrying out a purchase for example, we want to pay a low price while getting a high quality product. In the same manner, multiple objectives are present in most technical applications such as fast and energy efficient driving or designing light and stable constructions. This dilemma leads to the field of multiobjective optimization, where the aim is to minimize all relevant objectives simultaneously. While we are usually satisfied with one (global or even local) optimal solution in the single-objective setting, there generally exists an infinite number of *optimal compromises* in the situation where multiple objectives are present since the different objectives contradict each other.

In this talk multiobjective optimization problems are considered, where the constraints are given by elliptic or parabolic partial differential equation. The goal is to compute numerically Pareto optimal points for the underlying optimization problem. After discussing analytical aspects numerical solution methods are presented which utilize reduced-order modeling. A-posteriori error analysis ensures a desired accuracy for the Pareto optimal points. Numerical examples for different applications (including optimal control, parameter optimization and model predictive control) illustrate the theoretical findings and numerical solution approach. The presented work is partially supported by the DFG-SPP 1962.

The presented results are joint works with S. Banholzer, M. Bernreuther, M. Dellnitz, G. Fabrini, B. Gebken, L. Grüne, G. Müller and S. Peitz.

Optimal transport based regularization in PET
Benedikt Wirth
University of Münster

Abstract

I will present and analyse different variants of optimal transport that are amenable for use in imaging. Furthermore, as an exemplary application we will present a novel model for dynamic image reconstruction from positron emission tomography measurements. This is joint work with Bernhard Schmitzer and Klaus Schäfers.

Optimization of Phase-Field Damage Evolution

Winnifried Wollner
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Abstract

Within this talk, we will address optimization problems governed by time-discrete phase-field damage processes. The presence of an irreversibility of the fracture growth gives rise to a nonsmooth system of equations. To derive optimality conditions we introduce an additional regularization and show that the resulting optimization problem is well-posed.

To tackle discretization errors, as well as convergence in the limit of the irreversibility penalty, an improved differentiability result is shown for the time discrete regularized damage process.

Based upon this, we can show that certain local minimizers of the optimization problem can be approximated by the proposed penalty approach. Further, we will give a short discussion of resulting discretization error estimates.

This is joint work with R. Haller-Dintelmann, H. Meinlschmidt, M. Mohammadi, I. Neitzel, T. Wick

A Space-Time Finite Element Method for Spatio-temporally Sparse Optimal Control of Parabolic Equations

Huidong Yang
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Abstract

In this talk, we will present some numerical studies on some space-time Petrov–Galerkin finite element method for spatio-temporally sparse optimal control of parabolic equations with Dirichlet boundary conditions, and with box constraints on the control function. In particular, we consider parabolic optimal control problems, where the objective functional contains a Lipschitz continuous and convex but not Fréchet differentiable functional. The space-time finite element discretization of the optimality system, including both the state and adjoint state equations, relies on a Galerkin–Petrov variational formulation employing piecewise linear finite elements on unstructured simplicial space-time meshes.

The nonlinear optimality systems of equations are solved by the semismooth Newton method, whereas the linearized coupled state and adjoint state systems are solved by an algebraic multigrid preconditioned GMRES method.

This is a joint work with Ulrich Langer (RICAM), Olaf Steinbach (TU Graz) and Fredi Trööltzsch (TU Berlin).