Null Space Gradient Flows for Shape Optimization of Multiphysics Systems

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New trends in PDE constrained optimization
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Outline

1. Shape derivatives for a weakly coupled multiphysics system
2. Null space gradient flows for constrained optimization
3. Numerical illustrations on 2-d and 3-d test cases

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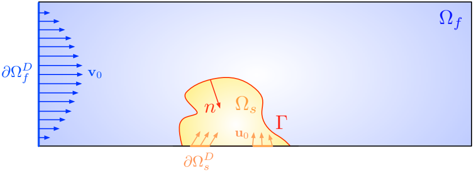
We are interested in multiphysics systems featuring

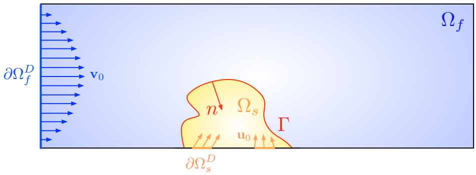
- fluids: velocity–pressure $(\boldsymbol{v}, p)$
- thermal exchanges: temperature field T, convected by $\boldsymbol{v}$
- mechanical structures: displacement $\boldsymbol{u}$, subjected to fluid-structure interaction with $\boldsymbol{v}$ and thermoelasticity with T.

1. Shape derivatives for a multiphysics system
Proposed system



- Incompressible Navier-Stokes equations for $(\boldsymbol{v}, p)$ in Ω_f

$$-\mathrm{div}(\sigma_f(\boldsymbol{v}, p)) + \rho\nabla\boldsymbol{v}\,\boldsymbol{v} = \boldsymbol{f}_f \text{ in } \Omega_f$$



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- Steady-state convection-diffusion for T_f and T_s in Ω_f and Ω_s:

$$\begin{aligned} -\mathrm{div}(k_f\nabla T_f) + \rho\boldsymbol{v}\cdot\nabla T_f &= Q_f \quad \text{in } \Omega_f \\ -\mathrm{div}(k_s\nabla T_s) &= Q_s \quad \text{in } \Omega_s \end{aligned}$$

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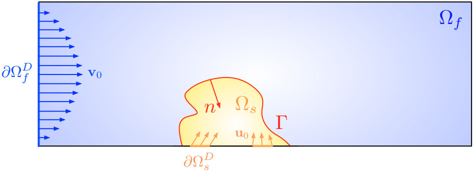

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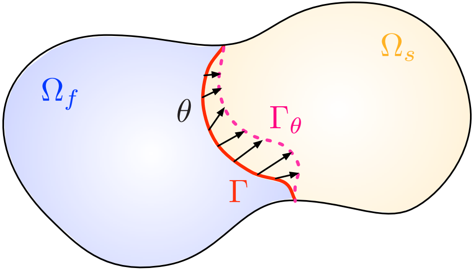
- Linearized thermoelasticity with fluid-structure interaction for $\boldsymbol{u}$ in Ω_s:

$$\begin{aligned} -\mathrm{div}(\sigma_s(\boldsymbol{u}, T_s)) &= \boldsymbol{f}_s && \text{in } \Omega_s \\ \sigma_s(\boldsymbol{u}, T_s) \cdot \boldsymbol{n} &= \sigma_f(\boldsymbol{v}, p) \cdot \boldsymbol{n} && \text{on } \Gamma. \end{aligned}$$

$$\min_{\Gamma} \; J(\Gamma)$$



1. Shape derivatives for a multiphysics system

The method of Hadamard

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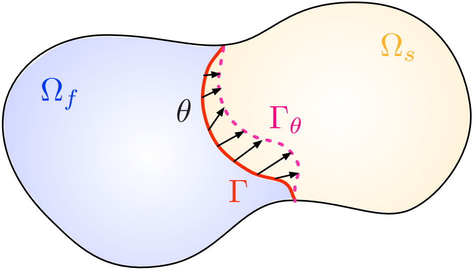

$$\Gamma_{\boldsymbol{\theta}} = (I + \boldsymbol{\theta})\Gamma, \text{ where } \boldsymbol{\theta} \in W_0^{1,\infty}(D, \mathbb{R}^d), \; ||\boldsymbol{\theta}||_{W^{1,\infty}(\mathbb{R}^d,\mathbb{R}^d)} < 1.$$

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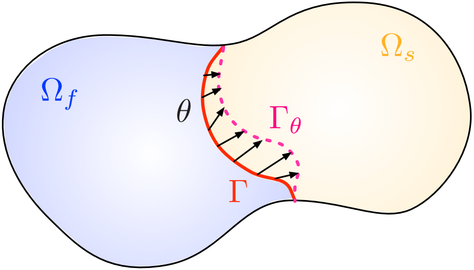

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$$J(\Gamma_{\boldsymbol{\theta}}) = J(\Gamma) + \frac{\mathrm{d}J}{\mathrm{d}\boldsymbol{\theta}}(\boldsymbol{\theta}) + o(\boldsymbol{\theta}), \quad \text{where } \frac{|o(\boldsymbol{\theta})|}{||\boldsymbol{\theta}||_{W^{1,\infty}(D,\mathbb{R}^d)}} \xrightarrow{\boldsymbol{\theta}\to 0} 0.$$

Proposition

Let $J(\Gamma, \boldsymbol{u}, T, \boldsymbol{v}, p)$ *an arbitrary functional with continuous partial derivatives and* $\boldsymbol{u}(\Gamma), T(\Gamma), \boldsymbol{v}(\Gamma), p(\Gamma)$ *the above state variables. Then, if these are smooth enough,* $\Gamma \mapsto J(\Gamma, \boldsymbol{u}(\Gamma), T(\Gamma), \boldsymbol{v}(\Gamma), p(\Gamma))$ *is shape differentiable and the derivative reads:*

Shape derivatives for a multiphysics system

Shape derivative of arbitrary functionals

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&\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}}\Big[J(\Gamma_{\boldsymbol{\theta}}, \boldsymbol{v}(\Gamma_{\boldsymbol{\theta}}), p(\Gamma_{\boldsymbol{\theta}}), T(\Gamma_{\boldsymbol{\theta}}), \boldsymbol{u}(\Gamma_{\boldsymbol{\theta}}))\Big](\boldsymbol{\theta}) \\
&\quad = \overline{\frac{\partial \mathfrak{J}}{\partial \boldsymbol{\theta}}}(\boldsymbol{\theta}) + \int_\Gamma (\boldsymbol{f}_f \cdot \boldsymbol{w} - \sigma_f(\boldsymbol{v}, p) : \nabla \boldsymbol{w} + \boldsymbol{n} \cdot \sigma_f(\boldsymbol{w}, q)\nabla \boldsymbol{v} \cdot \boldsymbol{n} + \boldsymbol{n} \cdot \sigma_f(\boldsymbol{v}, p)\nabla \boldsymbol{w} \cdot \boldsymbol{n})(\boldsymbol{\theta} \cdot \boldsymbol{n})\mathrm{d}s \\
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\end{aligned}
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$\mathfrak{J}$ is a "transported" functional:

$$\mathfrak{J}(\boldsymbol{\theta}, \hat{\boldsymbol{v}}, \hat{p}, \hat{T}, \hat{\boldsymbol{u}}) := J(\Gamma_{\boldsymbol{\theta}}, \hat{\boldsymbol{v}} \circ (I + \boldsymbol{\theta})^{-1}, \hat{p} \circ (I + \boldsymbol{\theta})^{-1}, \hat{T} \circ (I + \boldsymbol{\theta})^{-1}, \hat{\boldsymbol{u}} \circ (I + \boldsymbol{\theta})^{-1}).$$

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Partial derivative for J with respect to the shape.

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Three "adjoint" terms for each of the three physics.

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Adjoint variables $\boldsymbol{w}, q, S_f, S_s, \boldsymbol{r}$ are solved in a reversed cascade.

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2. Null space gradient flows for constrained optimization

- Our goal: solve constrained shape optimization problems

$$\begin{aligned} \min_{\Gamma} \quad & J(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma), T(\Gamma), \boldsymbol{u}(\Gamma)) \\ \text{s.t.} \quad & g_i(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma), T(\Gamma), \boldsymbol{u}(\Gamma)) = 0,\ 1 \leq i \leq p\cdot \\ & h_i(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma), T(\Gamma), \boldsymbol{u}(\Gamma)) \leq 0,\ 1 \leq i \leq q \end{aligned}$$

  with *arbitrary* functionals J, g_i, h_i;
- if possible, no fine tunings of optimization algorithm parameters;
- must deal with unfeasible initializations.

2. Null space gradient flows for constrained optimization

For the exposure of our method, let us consider

$$\min_{x\in V} \quad J(x)$$
$$\text{s.t.} \begin{cases} \boldsymbol{g}(x) = 0 \\ \boldsymbol{h}(x) \leq 0, \end{cases}$$

with

- $J : V \to \mathbb{R}$, $\boldsymbol{g} : V \to \mathbb{R}^p$ and $\boldsymbol{h} : V \to \mathbb{R}^q$ Fréchet differentiable

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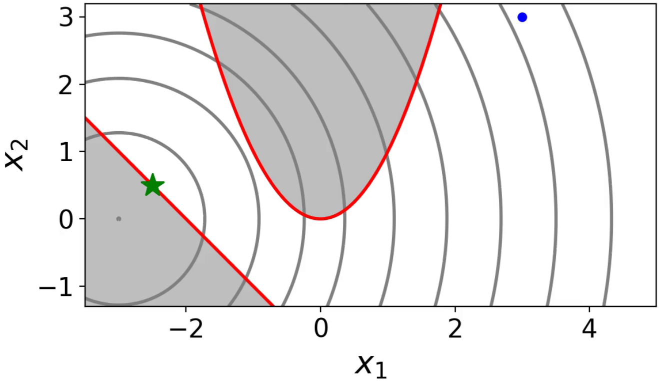
- V a Hilbert space equipped with a scalar product $(\cdot,\cdot)_V$.

2. Null space gradient flows for constrained optimization

$$\min_{(x_1,x_2)\in\mathbb{R}^2} \quad J(x_1,x_2) = x_1^2 + (x_2+3)^2$$

$$\text{s.t.} \quad \begin{cases} h_1(x_1,x_2) = -x_1^2 + x_2 & \leq 0 \\ h_2(x_1,x_2) = -x_1 - x_2 - 2 & \leq 0 \end{cases}$$



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- For equality constrained optimization, projected gradient flow (Tanabe (1980)):

$$\dot{x} = -(I - \mathrm{D}\boldsymbol{g}^T(\mathrm{D}\boldsymbol{g}\mathrm{D}\boldsymbol{g}^T)^{-1}\mathrm{D}\boldsymbol{g})\nabla J(x)$$

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$$\dot{x} = -\alpha_J(I - \mathrm{D}\boldsymbol{g}^T(\mathrm{D}\boldsymbol{g}\mathrm{D}\boldsymbol{g}^T)^{-1}\mathrm{D}\boldsymbol{g})\nabla J(x) - \alpha_C\mathrm{D}\boldsymbol{g}^T(\mathrm{D}\boldsymbol{g}\mathrm{D}\boldsymbol{g}^T)^{-1}\boldsymbol{g}(x)$$

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(gradient flow on $V = \{x \in V \,|\, \boldsymbol{g}(x) = 0\}$) Then Yamashita (1980) added a Gauss-Newton direction:

$$\dot{x} = -\alpha_J(I - \mathrm{D}\boldsymbol{g}^T(\mathrm{D}\boldsymbol{g}\mathrm{D}\boldsymbol{g}^T)^{-1}\mathrm{D}\boldsymbol{g})\nabla J(x) - \alpha_C \mathrm{D}\boldsymbol{g}^T(\mathrm{D}\boldsymbol{g}\mathrm{D}\boldsymbol{g}^T)^{-1}\boldsymbol{g}(x)$$

$\boldsymbol{g}(x(t)) = \boldsymbol{g}(x(0))e^{-\alpha_C t}$ and $J(x(t))$ decreases if $\boldsymbol{g}(x(t)) = 0$.

2. Null space gradient flows for constrained optimization

For *both* equality constraints $\boldsymbol{g}(x) = 0$ and inequality constraints $\boldsymbol{h}(x) \leq 0$, we consider:

$$\dot{x} = -\alpha_J \boldsymbol{\xi}_J(x(t)) - \alpha_C \boldsymbol{\xi}_C(x(t))$$

with

$$\boldsymbol{\xi}_J(x) := (I - \mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}^T (\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)} \mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}^T)^{-1} \mathrm{D}\boldsymbol{C}_{\widehat{I}(x)})(\nabla J(x))$$

$$\boldsymbol{\xi}_C(x) = \mathrm{D}\boldsymbol{C}_{\widetilde{I}(x)}^T (\mathrm{D}\boldsymbol{C}_{\widetilde{I}(x)} \mathrm{D}\boldsymbol{C}_{\widetilde{I}(x)}^T)^{-1} \boldsymbol{C}_{\widetilde{I}(x)}(x),$$

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$$\boldsymbol{\xi}_C(x) = \mathrm{D}\boldsymbol{C}^T_{\widetilde{I}(x)}(\mathrm{D}\boldsymbol{C}_{\widetilde{I}(x)}\mathrm{D}\boldsymbol{C}^T_{\widetilde{I}(x)})^{-1}\boldsymbol{C}_{\widetilde{I}(x)}(x),$$

$\widetilde{I}(x)$ the set of violated constraints:

$$\widetilde{I}(x) = \{i \in \{1, \ldots, q\} \mid h_i(x) \geqslant 0\}.$$

$$\boldsymbol{C}_{\widetilde{I}(x)} = \left[\boldsymbol{g}(x) \quad | \quad (h_i(x))_{i \in \widetilde{I}(x)}\right]^T$$

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$\widehat{I}(x) \subset \widetilde{I}(x)$ is an "optimal" subset of the active or violated constraints which can be computed by mean of a dual subproblem.

$$\widehat{I}(x) := \{i \in \widetilde{I}(x) \,|\, \mu_i^*(x) > 0\}.$$

$$\boldsymbol{C}_{\widehat{I}(x)} = \left[\boldsymbol{g}(x) \quad | \quad (h_i(x))_{i \in \widehat{I}(x)} \right]^T$$

Definition

Let $(\boldsymbol{\lambda}^*(x), \boldsymbol{\mu}^*(x)) \in \mathbb{R}^p \times \mathbb{R}^{\mathrm{Card}\widetilde{I}(x)}$ the solutions of the following dual minimization problem:

$$(\boldsymbol{\lambda}^*(x), \boldsymbol{\mu}^*(x)) := \arg \min_{\substack{\boldsymbol{\lambda}\in\mathbb{R}^p \\ \boldsymbol{\mu}\in\mathbb{R}^{\widetilde{q}(x)},\, \boldsymbol{\mu}\geqslant 0}} ||\nabla J(x) + \mathrm{D}\boldsymbol{g}(x)^T\boldsymbol{\lambda} + \mathrm{D}\boldsymbol{h}_{\widetilde{I}(x)}(x)^T\boldsymbol{\mu}||_V.$$

Define $\widehat{I}(x)$ the set obtained by collecting the non zero components of $\boldsymbol{\mu}^*(x)$:

$$\widehat{I}(x) := \{i \in \widetilde{I}(x) \,|\, \mu_i^*(x) > 0\}.$$

The best descent direction $-\boldsymbol{\xi}_J(x)$ must be proportional to

$$\boldsymbol{\xi}^* = \quad \arg\min_{\boldsymbol{\xi}\in V} \quad \mathrm{D}J(x)\boldsymbol{\xi}$$

$$\text{s.t.} \begin{cases} \mathrm{D}\boldsymbol{g}(x)\boldsymbol{\xi} = 0 \\ \mathrm{D}\boldsymbol{h}_{\widetilde{I}(x)}(x)\boldsymbol{\xi} \leq 0 \\ ||\boldsymbol{\xi}||_V \leq 1. \end{cases}$$

where $\boldsymbol{h}_{\widetilde{I}(x)}(x) = (h_i(x))_{i\in\widetilde{I}(x)}$

Proposition

$\boldsymbol{\xi}^*(x)$ is explicitly given by:

$$\boldsymbol{\xi}^*(x) = -\frac{\Pi_{\boldsymbol{C}_{\widehat{I}(x)}}(\nabla J(x))}{||\Pi_{\boldsymbol{C}_{\widehat{I}(x)}}(\nabla J(x))||_V},$$

with

$$\Pi_{\boldsymbol{C}_{\widehat{I}(x)}}(\nabla J(x)) = (I - \mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}^T(\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}^T)^{-1}\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)})(\nabla J(x))$$

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whence our definition

$$\dot{x} = -\alpha_J \boldsymbol{\xi}_J(x(t)) - \alpha_C \boldsymbol{\xi}_C(x(t))$$

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2. Null space gradient flows for constrained optimization

We can prove:

1. Constraints are asymptotically satisfied:

$$\boldsymbol{g}(x(t)) = e^{-\alpha_C t}\boldsymbol{g}(x(0)) \text{ and } \boldsymbol{h}_{\widetilde{I}(x(t))} \leq e^{-\alpha_C t}\boldsymbol{h}(x(0))$$

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3. All stationary points x^* of the ODE are KKT points

2. Null space gradient flows for constrained optimization

For shape optimization

$$\dot{x} = -\alpha_J \boldsymbol{\xi}_J(x(t)) - \alpha_C \boldsymbol{\xi}_C(x(t))$$

works the same with

$$\boldsymbol{\xi}_J(x) := (I - \mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}^{\mathcal{T}}(\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)}^{\mathcal{T}})^{-1}\mathrm{D}\boldsymbol{C}_{\widehat{I}(x)})(\nabla J(x))$$

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where the transpose $\mathcal{T}$ and the gradient ∇ must be computed with respect to $\langle , \rangle_V$ thanks to an identification problem.

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where the transpose $\mathcal{T}$ and the gradient ∇ must be computed with respect to $\langle , \rangle_V$ thanks to an identification problem.
This encompasses the celebrated regularization and extension step of the shape derivative in numerical algorithms.

Outline

1. Shape derivatives for a weakly coupled multiphysics system
2. Null space gradient flows for constrained optimization
3. Numerical illustrations on 2-d and 3-d test cases

3. Numerical applications

Lift-drag minimization:

$$\min \quad -\mathrm{Lift}(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma))$$

$$s.t. \begin{cases} \mathrm{Drag}(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma)) \leq \mathtt{DRAG}_0 \\ \mathrm{Vol}(\Omega_f) = V_0 \\ \boldsymbol{X}(\Omega_s) := \dfrac{1}{|\Omega_s|} \displaystyle\int_{\Omega_s} \boldsymbol{x} \,\mathrm{d}x = \boldsymbol{x}_0, \end{cases}$$

$$\mathrm{Lift}(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma)) := -\int_\Gamma \boldsymbol{e}_y \cdot \sigma_f(\boldsymbol{v}, p)\boldsymbol{n} \mathrm{d}s,$$

$$\mathrm{Drag}(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma)) := \int_{\Omega_f} \sigma_f(\boldsymbol{v}, p) : \nabla \boldsymbol{v} \mathrm{d}x.$$

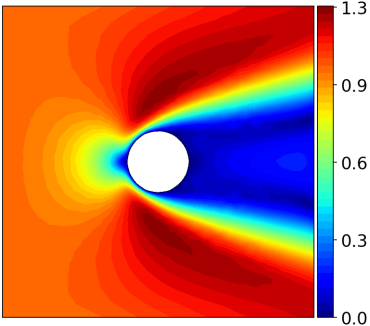

Figure: Optimized 2-d lift-drag flow profile.

Lift-drag minimization:

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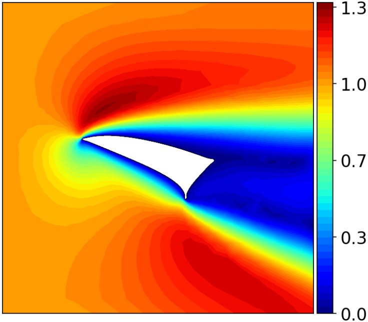
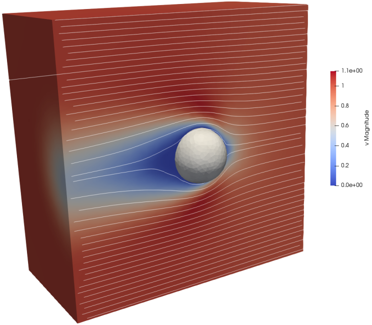


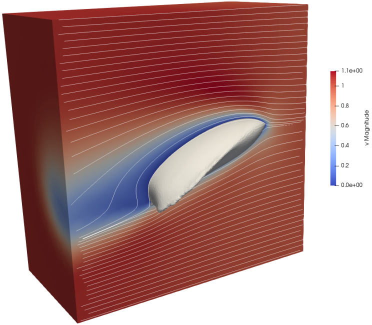
Figure: Optimized 2-d lift-drag flow profile.

3. Numerical applications

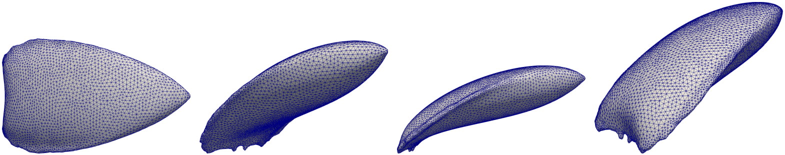
Lift-drag minimization in 3-d:



(a) Initial shape



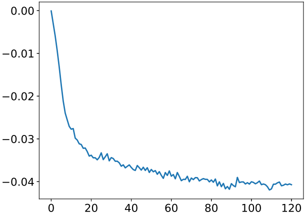
(b) Optimized design



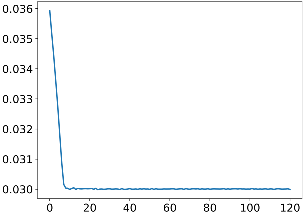
(c) Optimized design (other 3-d views)

3. Numerical applications

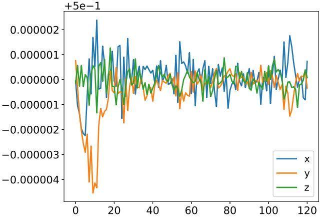
Lift-drag minimization in 3-d, convergence histories.



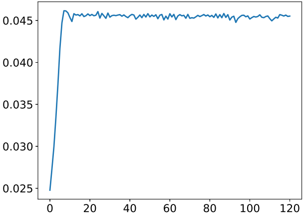
(a) Objective function.



(b) Volume constraint.



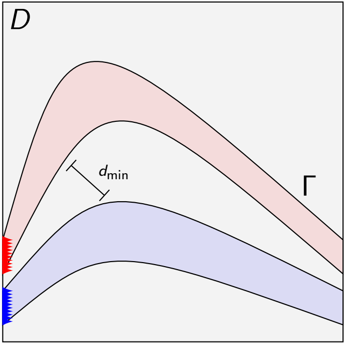
(c) Center of mass constraint.



(d) Drag constraint.

3. Numerical applications

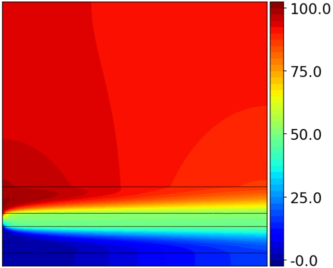
Bi-tube heat exchanger with non penetrating constraint



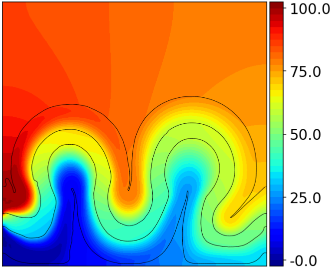
$$\min_{\Omega_f \subset D} \quad J(\Omega_f) = -\left(\int_{\Omega_{f,cold}} \rho c_p \boldsymbol{v} \cdot \nabla T \mathrm{d}x - \int_{\Omega_{f,hot}} \rho c_p \boldsymbol{v} \cdot \nabla T \mathrm{d}x \right)$$

$$s.t. \begin{cases} \mathrm{DP}(\Omega_f) = \int_{\partial\Omega_f^D} p \mathrm{d}s - \int_{\partial\Omega_f^N} p \mathrm{d}s \leq \mathrm{DP}_0 \\ Q_{hot \leftrightarrow cold}(\Omega_f) \geqslant d_{\min}. \end{cases}$$

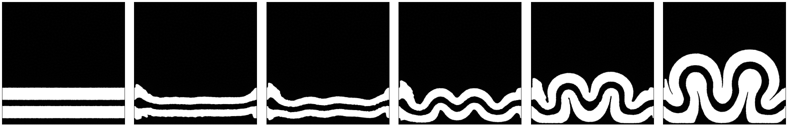
Bi-tube heat exchanger with non penetrating constraint



(a) Initial temperature field

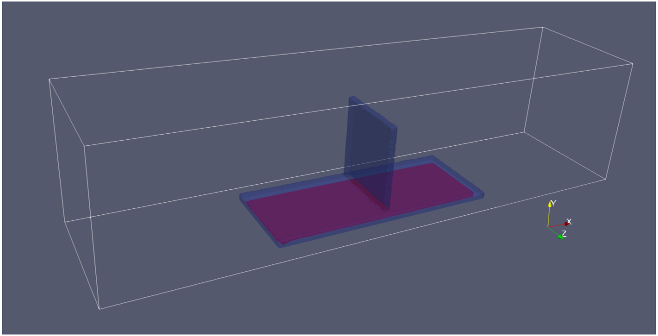


(b) Final temperature field.

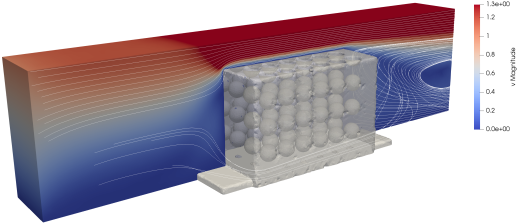


(c) Intermediate iterations

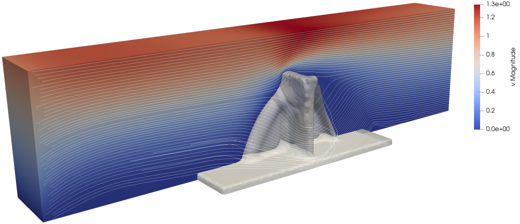
3-d compliance minimization problem with fluid-structure interaction



3-d compliance minimization problem with fluid-structure interaction



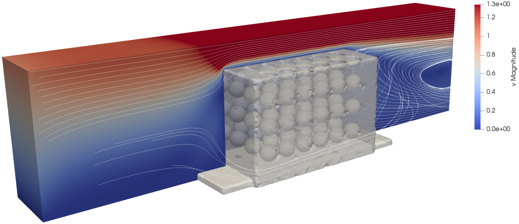
(a) Initial shape



(b) Final design

3. Numerical applications

3-d compliance minimization problem with fluid-structure interaction



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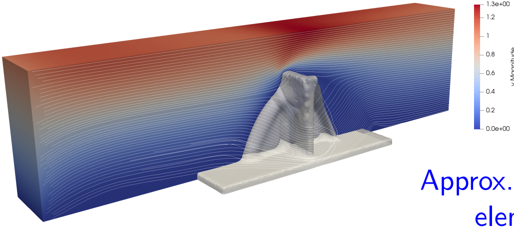
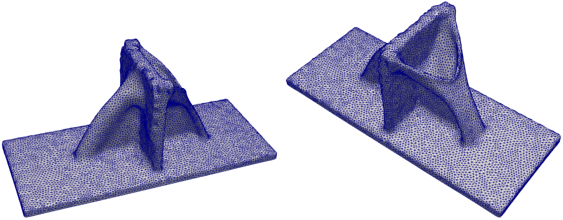


Approx. 2 millions elements.

(b) Final design



(a) Final design.

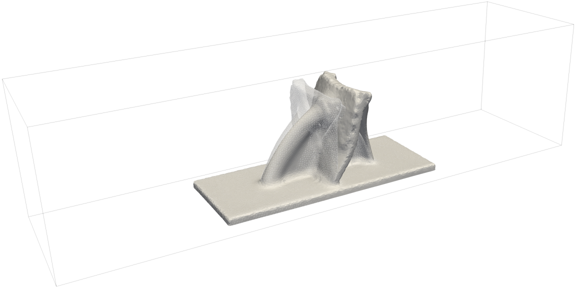


Figure: Linear elastic deformation.

3-d compliance minimization problem with fluid-structure interaction

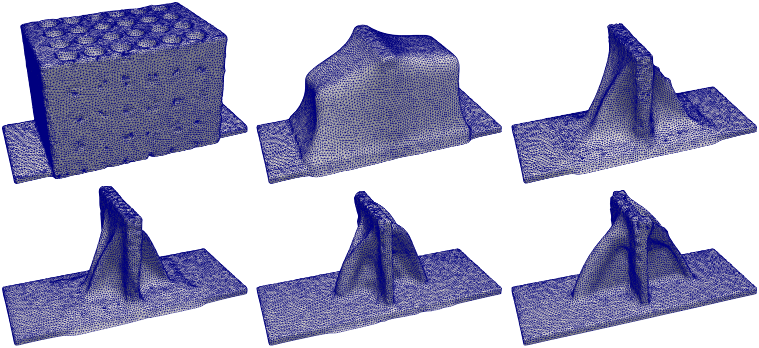


Figure: Intermediate iterations 0, 40, 100, 125, 175 and 300.

References

Feppon, F., Allaire, G., Bordeu, F., Cortial, J., and Dapogny, C. Shape optimization of a coupled thermal fluid-structure problem in a level set mesh evolution framework.
SeMA Journal (2019).

Feppon, F., Allaire, G., and Dapogny, C. Null space gradient flows for constrained optimization with applications to shape optimization.
HAL preprint hal-01972915 (2019).

Feppon, F., Allaire, G., and Dapogny, C. A variational formulation for computing shape derivatives of geometric constraints along rays.
HAL preprint hal-01879571 (2019).

Many thanks for your attention.