Additive manufacturing constraints in topology optimization of structures

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Outline

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- I - Introduction: a short review of shape and topology optimization of structures
- II - Additive manufacturing
- III - Mechanical constraint for overhang limitation
- IV - Thermal residual stresses
- V - Supports
- VI - Conclusion and perspectives

A "hot" topic with a lot of room for new ideas in modeling...

Sofia project: Add-Up, Michelin, Safran, ESI, etc.

Minimize an objective function $J(\Omega)$ over a set $\mathcal{U}_{ad}$ of admissibles shapes Ω (including possible topology changes)

$$\inf_{\Omega \in \mathcal{U}_{ad}, P(\Omega) \leq 0} J(\Omega)$$

with one or several constraints $P(\Omega)$

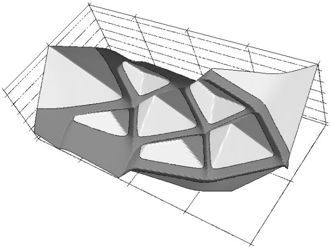
$$J(\Omega) = \int_{\Omega} j(u_{\Omega})\, dx\,, \quad P(\Omega) = \int_{\Omega} c(u_{\Omega})\, dx$$

where u_{Ω} is the solution of a partial differential equation (state equation)

$$PDE(u_{\Omega}) = 0 \quad \text{in} \quad \Omega$$

Here, the PDE is the system of linearized elasticity, $J(\Omega)$ is the compliance and one first constraint is the weight.

Level set method (Osher and Sethian)



A shape $\Omega \subset D$ is parametrized by a **level set** function

$$\psi(x) < 0 \Leftrightarrow x \in \Omega, \ \ \psi(x) > 0 \Leftrightarrow x \in (D \setminus \Omega)$$

Assume that the shape $\Omega(t)$ evolves in time t with a normal velocity $V(t,x)$. Then its motion is governed by the following Hamilton Jacobi equation

$$\frac{\partial \psi}{\partial t} + V|\nabla_x \psi| = 0 \quad \text{in } D.$$

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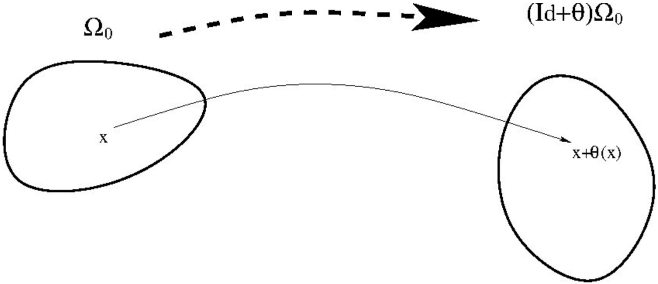
The velocity V is deduced from the shape gradient of the objective function.

To compute shape gradients we rely on Hadamard's method.

Let Ω_0 be a reference domain. Shapes are parametrized by a **vector field** θ:

$$\Omega = (\,\mathrm{Id} + \theta)\Omega_0 \quad \text{with} \quad \theta \in C^1(\mathbb{R}^d;\mathbb{R}^d).$$



Definition: the shape derivative of $J(\Omega)$ at Ω_0 is the Fréchet differential of $\theta \to J\big((\mathrm{Id} + \theta)\Omega_0\big)$ at 0.

Hadamard structure theorem: the shape derivative of $J(\Omega)$ can always be written (in a distributional sense)

$$J'(\Omega_0)(\theta) = \int_{\partial\Omega_0} \theta(x) \cdot n(x)\, j(x)\, ds$$

where $j(x)$ is an integrand depending on the state u and an adjoint p.

The normal velocity $V = \theta \cdot n$ is chosen so that $J'(\Omega_0)(\theta) \le 0$.
Simplest choice: $V = \theta \cdot n = -j$ (but other ones are possible).

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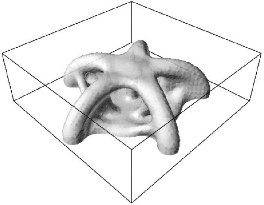
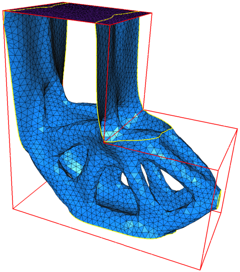
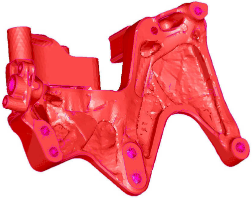
1. Initialization of the level set function ψ_0 (including holes).
2. Iteration until convergence for $k \geq 1$:
   1. Compute the elastic displacement u_k for the shape ψ_k. Deduce the shape gradient = normal velocity = V_k
   2. Advect the shape with V_k (solving the Hamilton Jacobi equation) to obtain a new shape ψ_{k+1}.

Optimization algorithms:

1. Lagrangian (possibly augmented) algorithm,
2. SLP (sequential linear programming).

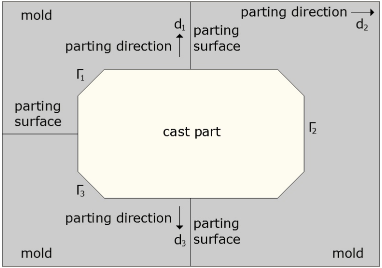
⋆ compliance minimization with a weight constraint

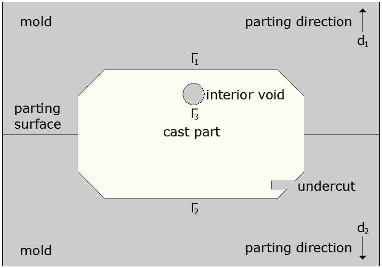




▸ dome ▸ L-beam ▸ motor-support

⋆ very hard to manufacture !

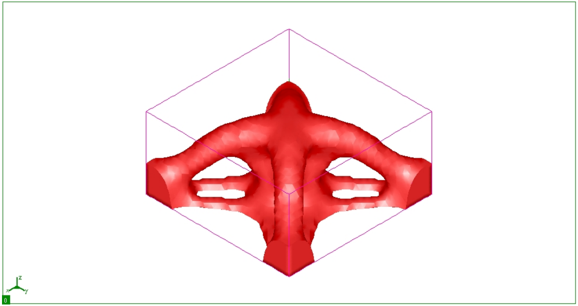
The molds and cast part should not be broken under removal: castable (left), non-castable (right).

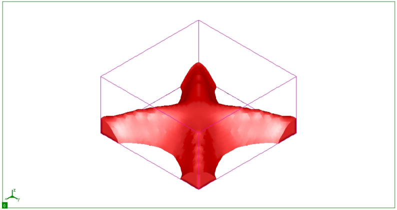


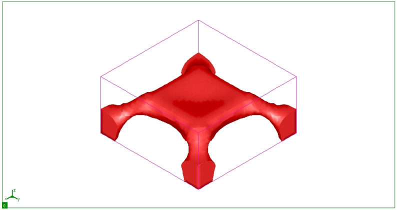


Delicate to implement in an optimization algorithm...

No mold (top), 1 mold (left b.) 2 molds (right b.)

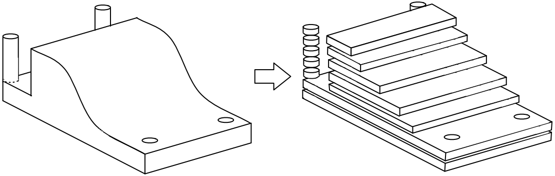




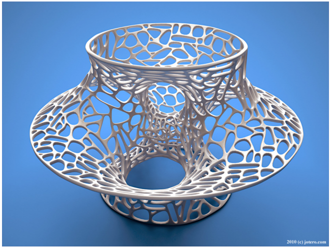


II - Additive Manufacturing

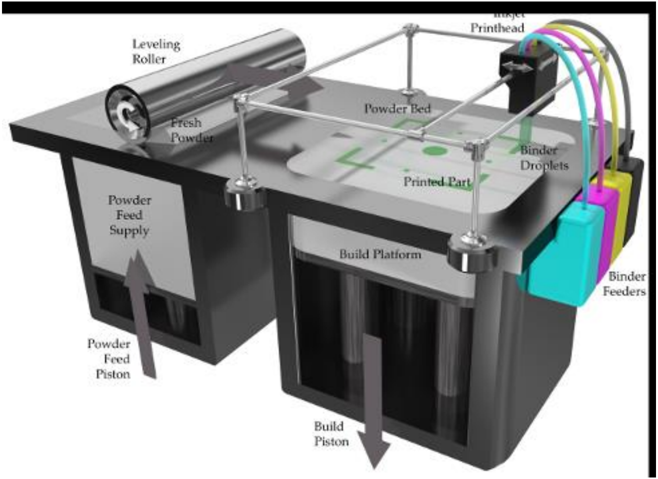
- Structures built layer by layer



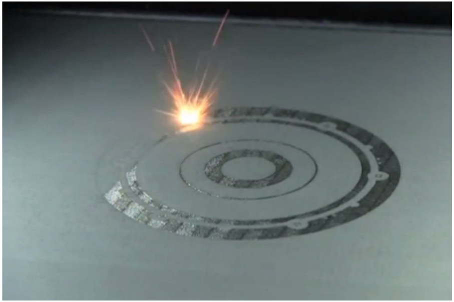
- No topological constraints on the built structures

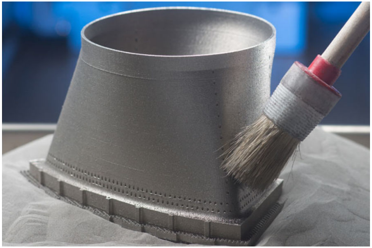


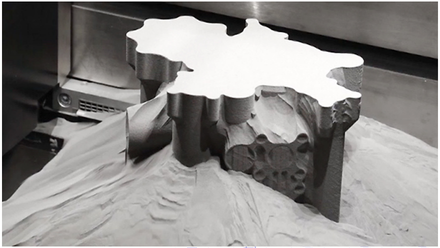
Metallic powder melted by a laser or an electron beam.



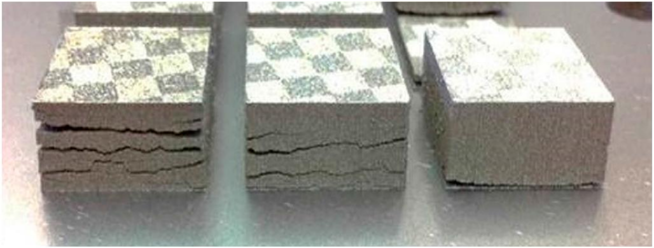
Metallic additive manufacturing

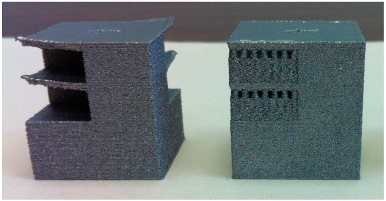




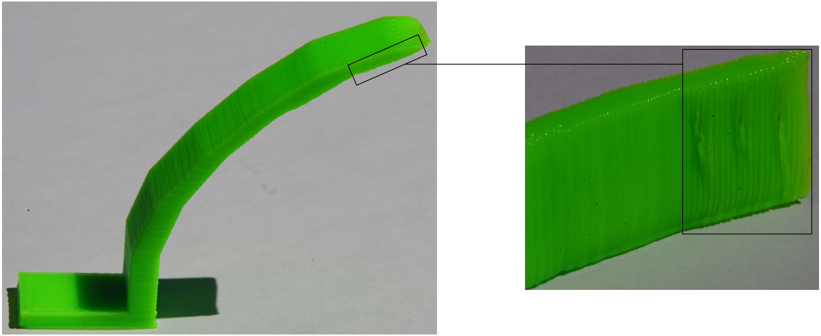


Some failures...





Overhang limitation



The angle between the structural boundary and the build direction has an impact on the quality of the processed shape.

Overhang limitation

Example of a bad 3-d printing due to overhangs.

Additive Manufacturing: Difficulties

→ no constraints related to topology, but...

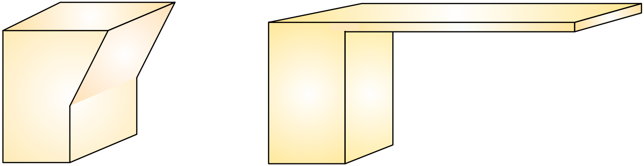
→ constraints related to the fabrication process !

- almost horizontal **overhang** surfaces cannot be built
- metal melting → large temperatures → **thermal residual stresses** and thermal deformations
- deformations of the structure may stop the powder deposition system
- adding (and removing) **supports**,
- preferred orientation of thin and slender structures,
- minimal time (or energy) for completion,
- removing the powder (no closed holes).

Why a mechanical constraint ?

Because geometrical constraint fail most of the time !

A naive idea: geometric constraint on the normal angle



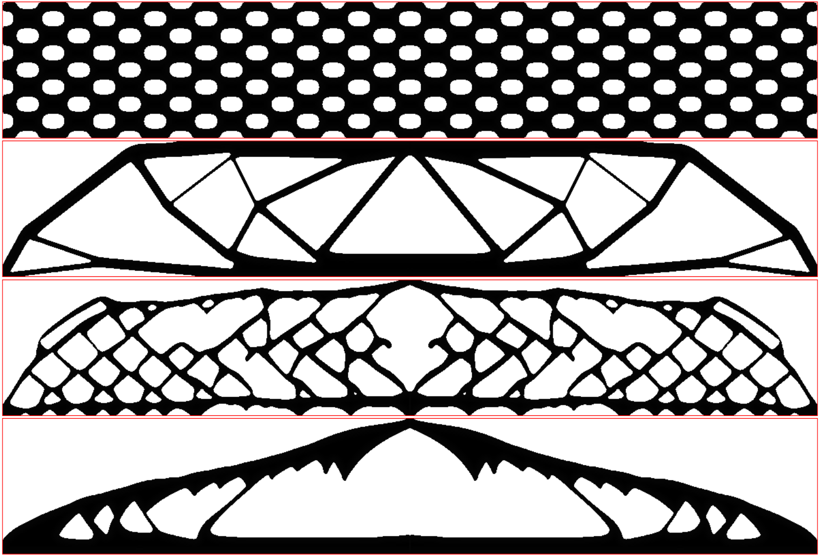
To avoid bad 3-d printing due to overhangs, small angles of the normal to the shape with the build direction d are forbidden.

For a given angle ϕ, our pointwise criterion reads

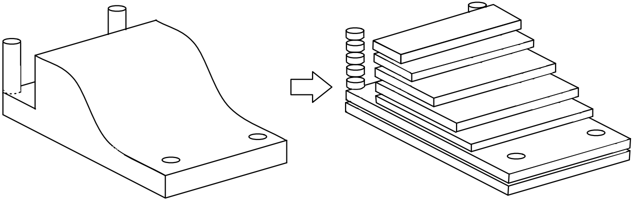
$$n(x) \cdot d \leq \cos\phi, \qquad \forall x \in \partial\Omega.$$

See: Leary et al. (2014), Gaynor and Guest (2016), Langelaar (2016, 2017), Allaire et al. (JCP 2017).

Dripping effect

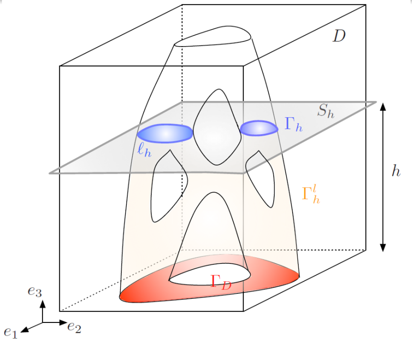


New approach: layer by layer modeling



Additive manufacturing involves a layer by layer process.

We model this process with a mechanical approach.



For a final shape Ω, define **intermediate shapes** Ω_i of increasing height h_i

$$\Omega_i = \{x \in \Omega \text{ such that } x_d \leq h_i\} \quad 1 \leq i \leq n.$$

Two different state equations:

1. for the objective function of the final shape Ω,
2. for the additive manufacturing constraint on each Ω_i.

1st state equation for the final shape

For a given applied load $f : \Gamma_N \to \mathbb{R}^d$,

$$\begin{cases} -\operatorname{div}(A\, e(u)) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_D \\ \big(A\, e(u)\big) n = f & \text{on } \Gamma_N \\ \big(A\, e(u)\big) n = 0 & \text{on } \Gamma \end{cases}$$

Objective function: compliance

$$J(\Omega) = \int_{\Gamma_N} f \cdot u \, dx,$$

Apply self-weight (gravity g) to the top layer of intermediate shapes Ω_i:

$$\begin{cases} -\mathrm{div}\,(A\,e(u_i)) & = & \rho g_\delta & \text{in } \Omega_i, \\ u_i & = & 0 & \text{on } \Gamma_D, \\ \big(A\,e(u_i)\big)\,n & = & 0 & \text{on } \Gamma_i, \end{cases}$$

$$\text{with} \qquad g_\delta(x) = \begin{cases} g & \text{if } h_i - \delta < x_d < h_i, \\ 0 & \text{otherwise,} \end{cases}$$

The boundary conditions are different from the first state equation.
Total self-weight compliance constraint:

$$P(\Omega) = \sum_{i=1}^{n} \int_{\Omega_i} Ae(u_i) : e(u_i)\,dx = \sum_{i=1}^{n} \int_{\Omega_i} \rho g_\delta \cdot u_i\,dx$$

We solve the optimization problem:

$$\begin{array}{rl} \min\limits_{\Omega\subset D} & J(\Omega) \\ \text{s.t.} & V(\Omega) \leq 0.20|D| \\ & P(\Omega) \leq \alpha P(\Omega_{ref}) \end{array}$$

where Ω_{ref} is the optimal design without constraint and $\alpha \in (0,1)$ is a parameter of the method.

Recall that $J(\Omega)$ is the compliance for the final shape, $V(\Omega)$ is the volume and $P(\Omega)$ is the self-weight constraint for the intermediate shapes.

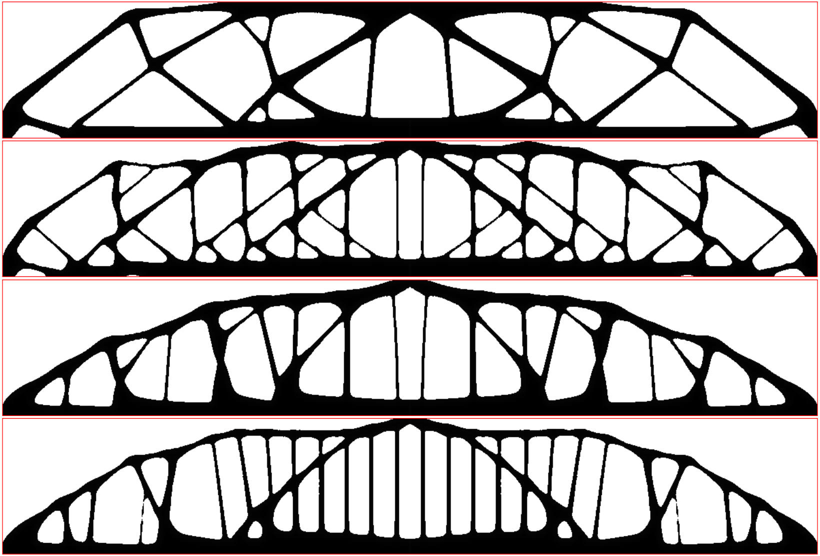
Some subtle issues in the shape derivation of $P(\Omega)$...

More details can be found in:

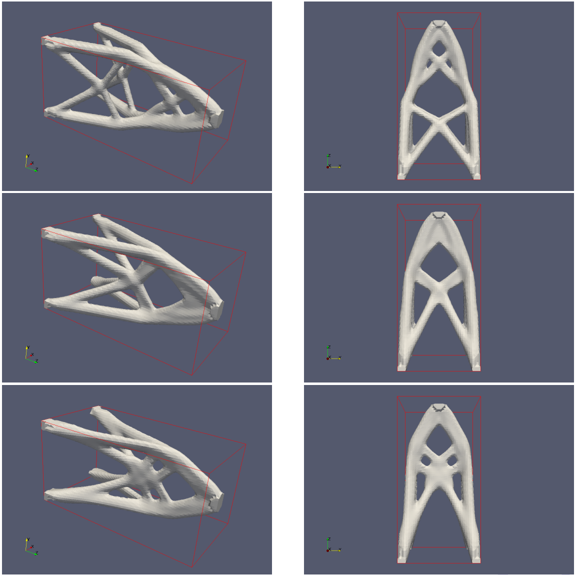
G. Allaire, Ch. Dapogny, A. Faure, G. Michailidis, *Shape optimization of a layer by layer mechanical constraint for additive manufacturing*, C. R. Math. Acad. Sci. Paris, 355, no. 6, 699-717 (2017).

G. Allaire, C. Dapogny, R. Estevez, A. Faure and G. Michailidis, *Structural optimization under overhang constraints imposed by additive manufacturing technologies*, J. Comput. Phys. 351, pp.295-328 (2017).

Self-weight compliance constraint (varying α)



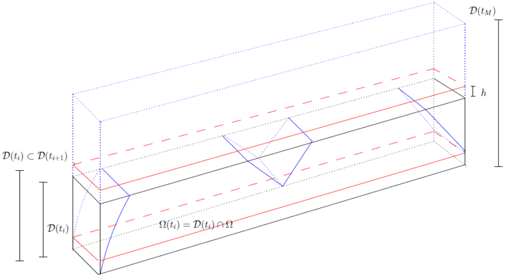
Self-weight compliance constraint in 3-d



Same idea with a more involved model:

- Keep intermediate **"layer by layer"** shapes $(\Omega_i)_{i=1,\dots,n}$.
- Each layer i is built between time t_{i-1} and t_i.
- Holes are now **filled by a metallic powder**.
- Thermal residual stress computed by a model as in
  L. Van Belle, J.-C. Boyer, G. Vansteenkiste, *Investigation of residual stresses induced during the selective laser melting process*, Key Engineering Materials, 1828-2834 (2013).
  M. Megahed, H.-W. Mindt, N. NâDri, H. Duan, O. Desmaison, *Metal additive-manufacturing process and residual stress modeling*, Integrating Materials and Manufacturing Innovation, 5:4, (2016).

Notations



1. Each layer i is built between time t_{i-1} and t_i, $1 \leq i \leq n$.
2. Build chamber D, vertical build direction e_d.
3. Intermediate domains $D_i = \{x \in D \text{ such that } x_d \leq h_i\}$.
4. Final shape Ω and intermediate shapes $\Omega_i = \Omega \cap D_i$.
5. Mixture $D_i = \Omega_i \cup P_i$ of solid and powder.

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Heat equation:

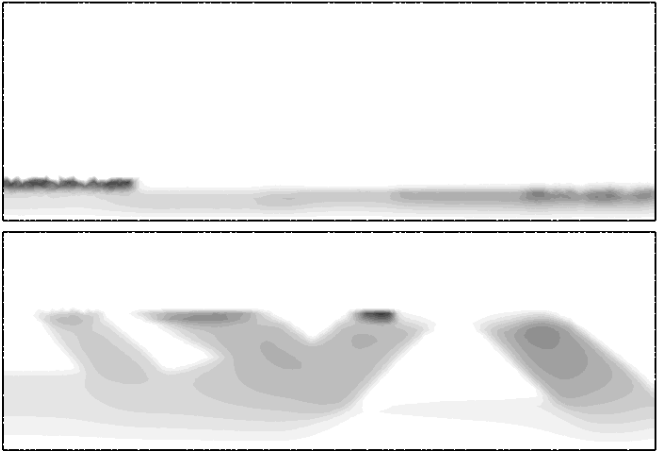
$$\begin{cases} \rho \dfrac{\partial T}{\partial t} - \mathrm{div}(\lambda \nabla T) = Q(t) & \text{in } (t_{i-1}, t_i) \times D_i \\ T = T_{init} & \text{on } (t_{i-1}, t_i) \times \Gamma_{base} \\ \lambda \nabla T \cdot n = -H_e(T - T_{init}) & \text{on } (t_{i-1}, t_i) \times (\partial D_i \setminus \Gamma_{base}) \\ T(t = t_{i-1}) = T_{init} & \text{in } D_i \setminus D_{i-1} \end{cases}$$

Thermoelastic quasi-static equation:

$$\begin{cases} -\mathrm{div}(\sigma) = 0 & \text{and } \sigma = \sigma^{el} + \sigma^{th} \quad \text{in } (t_{i-1}, t_i) \times D_i, \\ \sigma^{el} = Ae(u) & \text{and } \sigma^{th} = K(T - T_{init})\,\mathrm{Id}, \end{cases}$$

Material parameters ρ, λ, A, K are different for solid or powder.
Source term $Q(t)$ = beam spot, traveling on the upper layer.
Weak coupling: **first**, solve the heat equation, **second**, thermoelasticity.

Path of the source term $Q(t)$



▸ path

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The objective (or constraint) function is

$$J(\Omega) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_{D_i} j(u, \sigma, T)\, dx\, dt$$

where (u, σ, T) is the displacement, stress and temperature fields for the **intermediate shapes**. A constraint on the compliance of the final shape is imposed

$$C(\Omega) = \int_{\Omega} f \cdot u_{final}\, dx \leq C(\Omega_{ref}),$$

where u_{final} is the elastic displacement for the **final shape**, solution of

$$-\operatorname{div}(A\, e(u_{final})) = f \quad \text{in } \Omega$$

The shape derivative of $J(\Omega)$ is computed by an adjoint method.

Adjoint problems

Example for an objective $j(u)$ (without T and σ for simplicity).

Elasticity adjoint equation: no "backward effect"

$$-\operatorname{div}(e(\eta)) = -j'(u) \quad \text{in } (t_{i-1}, t_i) \times D_i$$

Adjoint heat equation: backward in time, from $i = n$ to 1,

$$\begin{cases} \rho \dfrac{\partial p}{\partial t} + \operatorname{div}(\lambda \nabla p) = K \operatorname{div} \eta & \text{in } (t_{i-1}, t_i) \times D_i \\ p = 0 & \text{on } (t_{i-1}, t_i) \times \Gamma_{base} \\ \lambda \nabla p \cdot n = -H_e p & \text{on } (t_{i-1}, t_i) \times (\partial D_i \setminus \Gamma_{base}) \\ p(t = t_n) = 0 & \text{in } D_n \end{cases}$$

Reversed order of coupling: **first**, solve the adjoint elasticity, **second**, the adjoint heat equation.

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1. Minimize the deviatoric part of the stress $\sigma_D = 2\mu e(u)_D$

$$J_1(\Omega) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_{D_i} |\sigma_D|^2 \, dx \, dt$$

2. Minimize the top vertical displacement (to allow the rake or roller to coat a new powder layer)

$$J_2(\Omega) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_{D_i \setminus D_{i-1}} \left| \max(0, u \cdot e_d - u_{max}) \right|^2 \, dx \, dt$$

Simplified model

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Ideas:

1. forget the layer by layer construction,
2. forget the moving source term.

Consequences:

- apply the thermo-mechanical model only at the final shape Ω,
- take a source term $Q(t, x)$ constant in time and in the solid (zero in the powder),
- perform just a few time steps,
- simpler and faster ! (More simplification are possible...)

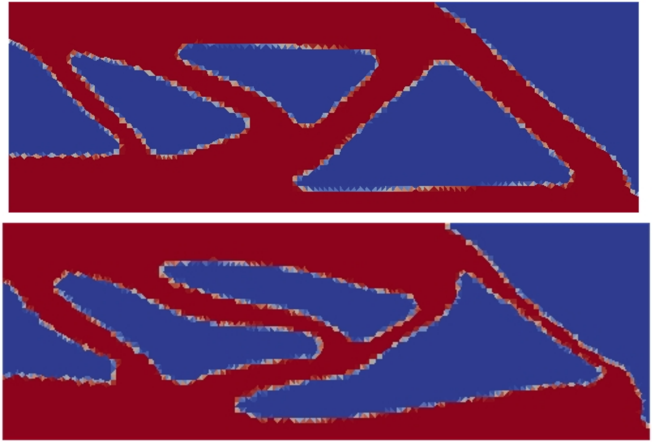
Test case 1

- Half MBB beam (2-d).
- Simplified model with 5 time steps.
- Minimize the vertical displacement (to allow the rake or roller to coat a new powder layer)

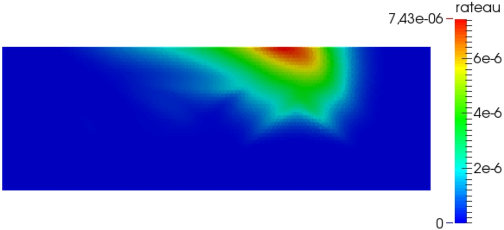
$$J_2(\Omega) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_D \left|\max(0, u \cdot e_d - u_{max})\right|^2 \, dx \, dt$$

- The value u_{max} is guessed from the initial design.
- Constraints on volume (fixed) and compliance.
- Initial design: optimal design for compliance minimization.

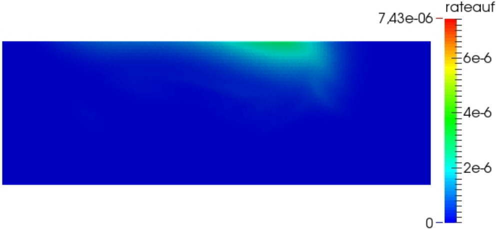
Initial (top) and final (bottom) shape



Initial and final constraint on the vertical displacement



initial



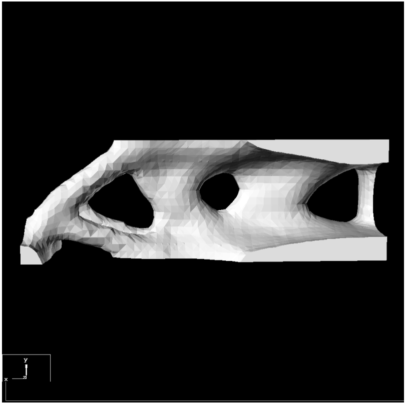
final

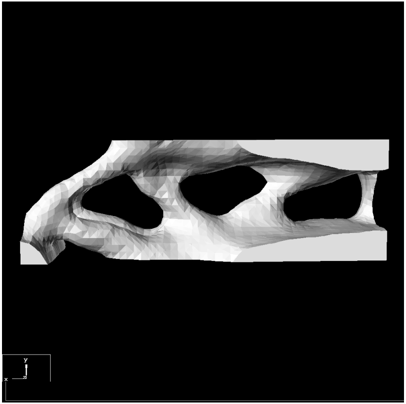
- Half MBB beam (3-d).
- Simplified model with 10 time steps.
- Minimize the vertical displacement (to allow the rake or roller to coat a new powder layer)

$$J_2(\Omega) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_D \left|\max(0, u \cdot e_d - u_{max})\right|^2 \, dx \, dt$$

- Constraints on volume (fixed) and compliance.
- Initial design: optimal design for compliance minimization.

Initial (left) and final (right) shapes





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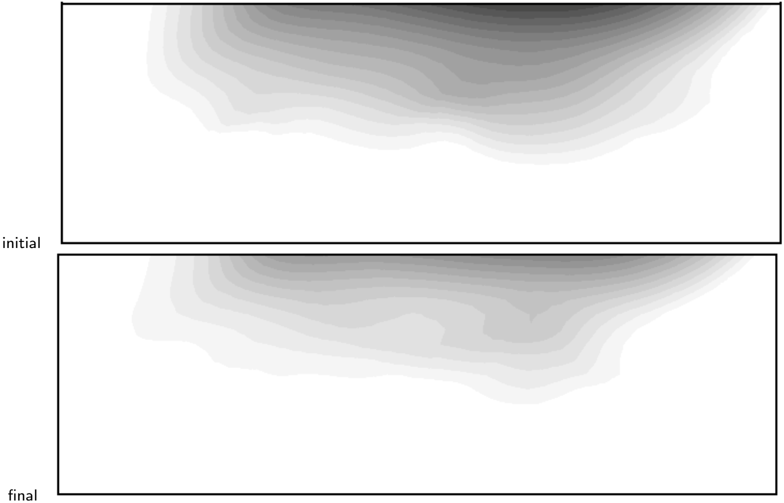
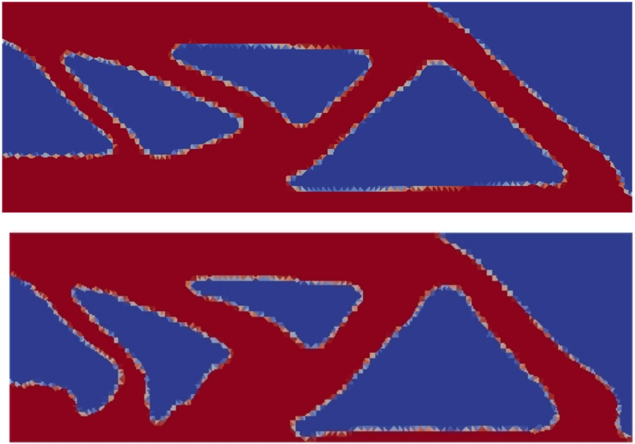


- Half MBB beam (2-d).
- **Full model with 20 layers and 5 time steps per layer.**
- Minimize the deviatoric part of the stress $\sigma_D = 2\mu e(u)_D$

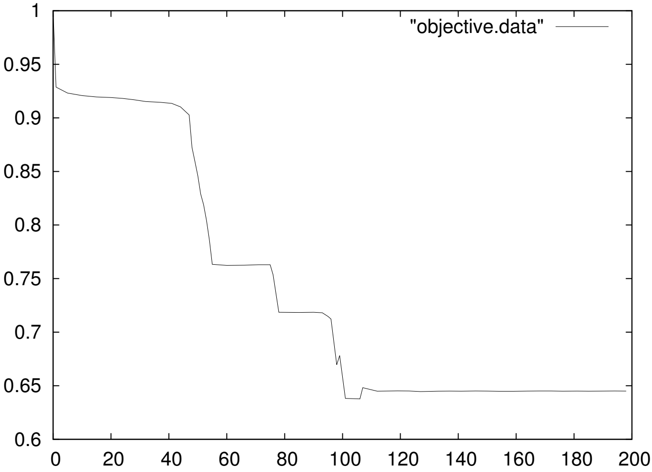
$$J_1(\Omega) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_D |\sigma_D|^2 \, dx \, dt$$

- The value u_{max} is guessed from the initial design.
- Constraints on volume (fixed) and compliance.
- Initial design: optimal design for compliance minimization.

Initial (top) and final (bottom) shape



Convergence history (thermal stress)



Plot of thermal stress $\sqrt{\int_0^T |\sigma^D|^2(x)\,dt}$

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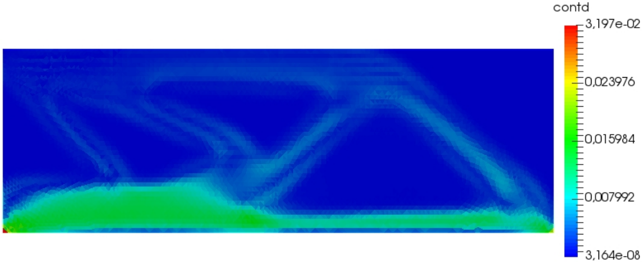
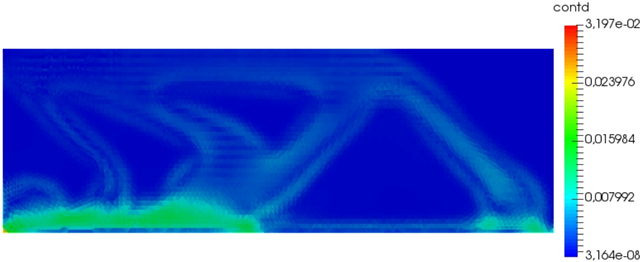


initial



final

More details in :

G. Allaire, L. Jakabcin, *Taking into account thermal residual stresses in topology optimization of structures built by additive manufacturing,* M3AS 28(12), 2313-2366 (2018).

V - Support optimization

- Support are scaffolds for inclined surfaces.
- Supports fix the shape to the baseplate.

Drawbacks

Impression time, additional material consumption, post-processing (removal)

- Support are scaffolds for inclined surfaces.
- Supports fix the shape to the baseplate.

Optimization goals

Given a certain design, to insure its successful 3D printing, optimize the topology of supports with minimal volume.

Bibliography on support optimization

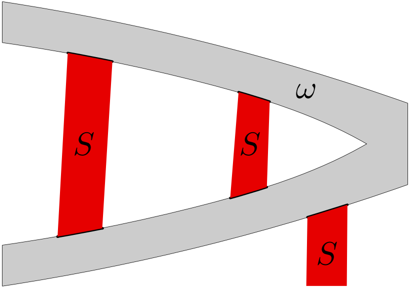
Many works !

Allaire et al., C. R. Math. Acad. Sci. Paris (2017), Cacace et al., Appl. Math. Model. (2017), Calignano, Materials & Design (2014), Dumas et al., ACM Trans. Graph. (2014), Gaynor and Guest, SMO (2016), Hu et al., Computer-Aided Design (2015), Kuo et al., SMO (2018), Langelaar, Additive Manufacturing (2016), Leary et al., Materials & Design (2014), Mirzendehdel and Suresh, Computer-Aided Design (2016), Qian, J. Num. Meth. Eng. (2017), Strano et al., Int. J. Adv. Manufact. Techn. (2013), Vanek et al., Computer Graphics Forum (2014), etc.

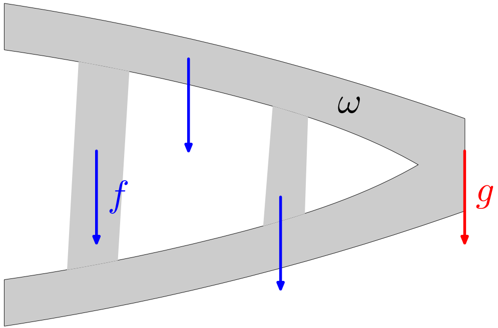
Support optimization or self-supported structure optimization.

Topology optimization of supports

- design domain D
- given structure $\omega \subset D$ (to be printed and non-optimizable)
- supports are denoted by $S \subset D$



Linearized elasticity with gravity loads in ω and S:



Simple model to mimick the effect of overhangs.

Linearized elasticity with gravity loads in ω and S:

$$e(u) = \frac{1}{2}(\nabla u + \nabla^t u) \quad \text{and} \quad \sigma = 2\mu e(u) + \lambda \mathrm{tr}(e(u))$$

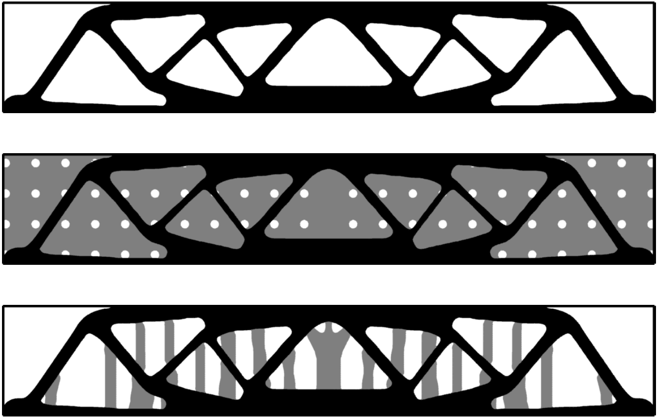
μ and λ may be different in ω and S (which can be some homogenized lattice material)

$$\begin{cases} -\operatorname{div}\sigma &= g(\rho_\omega \chi_\omega + \rho_S \chi_S) & \omega \cup S \\ u &= 0 & \Gamma_D \\ \sigma.n &= 0 & \Gamma_N \end{cases}$$

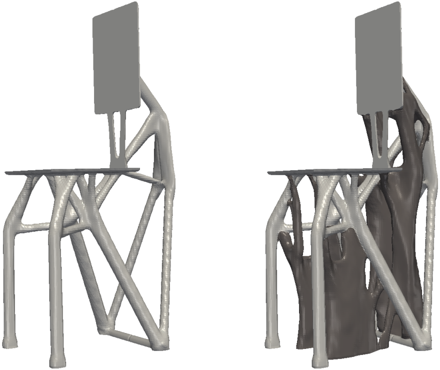
Compliance minimization (with volume constraint):

$$\min \mathcal{F}(S) = \int_{\omega \cup S} g(\rho_\omega \chi_\omega + \rho_S \chi_S) \cdot u$$

MBB beam in 2D (supports in grey)



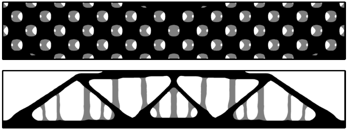
Supports for a 3D chair



▸ Rotation ▸ Optimization

- at every iteration we solve **two** state equations : one for the final loads on the structure ω alone and another for the building loads on the supported structure $S \cup \omega$
- evolve the two shapes simultaneously using two level set functions for the parametrization
- different shape derivatives on $\partial\omega \setminus S, \partial S \setminus \omega$ and $\partial\omega \cap \partial S$

The MBB example: video



VI - Conclusions and perspectives

- No limits for modeling ! Many possible variants...
- Ongoing work on support optimization
  G. Allaire, B. Bogosel, *Optimizing supports for additive manufacturing*, SMO 58(6), 2493-2515 (2018).
  G. Allaire, M. Bihr, B. Bogosel, *Support optimization in additive manufacturing for geometric and thermo-mechanical constraints*, submitted.
- Real experiments on building such structures.

Other topics in additive manufaturing

- optimal control of the laser path (Mathilde Boissier)
- lattice materials (Perle Geoffroy-Donders, Alex Ferrer)
- multi-physics optimization (Florian Feppon)