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Multirate DAE-Simulation and its Application in System Simulation Software for the Development of Electric Vehicles

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Abstract This work is devoted to the efficient simulation of large multi-physical networks stemming from automated modeling processes in system simulation software. The simulation of hybrid, battery and fuel cell electric vehicle applications requires the coupling of electric, mechanic, fluid and thermal networks. Each network is established by combining the connection structure of a graph with physical equations of elementary components and resulting in a differential algebraic equation (DAE). In order to speed up the simulation a non-iterative multirate time integration co-simulation method for the system of coupled DAEs is established. The power of the multirate method is shown via two representative examples of a battery powered electric vehicle with a cooling system for the battery pack and a three phase inverter with a cooling system.

1 Background and Introduction

State-of-the-art modeling and simulation packages such as AVL CRUISE$^{\text{TM}}$M[1], Dymola[2], or Amesim[3] offer many concepts for the automatic generation of dy-

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[1] https://www.avl.com/de/cruise-m

[2] http://www.dynasim.com

[3] http://www.plm.automation.siemens.com

namic system models. Modeling is done in a modularized way, based on a network of subsystems which again consists of simple standardized subcomponents. For instance, in case of HEVs (hybrid electric vehicles), BEVs (battery electric vehicles) and FCEVs (fuel cell electric vehicles) these can be the vehicle chassis, the drive line, the air path of the ICE (internal combustion engine) including combustion and exhaust aftertreatment, the cooling and lubrication system of the ICE and battery packs, the electrical propulsion system including the engine and a battery pack, the air conditioning and passenger cabin models, waste heat recovery and finally according control systems. Due to the complex interaction of the subsystems, the challenges in the development of future power trains do not only lie in the design of individual components but in the assessment of the power train as a whole. On a system engineering level it is required to optimize individual components globally and to balance the interaction of different subsystems. Due to the increasing complexity of the models, the systems exhibit largely varying time scales and are difficult in the numerical handle. A mainly automatized multirate approach is a promising way to decrease the computational effort.

The structure of the work is the following: In Section 2 the individual physical networks are introduced and the coupling conditions are stated in order to obtain a fully coupled system of network DAEs. The multirate time integration technique for the coupled system of network DAEs is described in Section 3 and the corresponding numerical results are presented in Section 4 and Section 5 and finally we conclude in Section 6.

2 Problem Formulation

We consider a network that is composed of multi-physical elements. The network elements describing the electric contribution are given by current sources, voltage sources, nodes, ground, resistors, capacitors and inductors. The fluid network consists of pipes, pumps, demands, junctions and reservoirs. The electro-thermal coupling is established by lumped mass elements representing the pipe wall and the masses from the battery and heat transfer connections. The individual components are assembled to a network $\mathscr{N}$, which is represented by a linear directed graph. The graph structure is described by an incidence matrix matrix A, which can be used for the model descriptions, cf. [7]. In the following we state the DAEs for the three main involved physical networks.

Electric Network

We consider an electric network $\mathscr{N}_E = \{R,C,L,V,I,N,G,B\}$ that is composed of resistors R, capacitors C, inductors L, voltage sources V, current sources I, nodes N, grounds G and batteries B. The DAE for the network in $\mathscr{N}_E$ in input-output form is given by: For given continuous inputs $(u_R^T, u_C^T, u_B^T)^T$ find the potentials $e = (e_N^T, e_G^T)^T$, the currents $j = (j_R^T, j_C^T, j_L^T, j_V^T, j_B^T)^T$ and the outputs $y = (y_R^T)^T$, such that

$$
\begin{aligned}
A_R j_R + A_C j_C + A_L j_L + A_V j_V + A_V j_B + A_I \bar{j}_I &= 0\\
r(u_R) j_R - A_R^T e &= 0\\
j_C - \frac{d(c(u_C)A_C^T e)}{dt} &= 0\\
l \frac{d j_L}{dt} - A_L^T e &= 0\\
A_V^T e &= \bar{v}_V\\
A_B^T e &= \bar{v}_B(j_B, u_B)\\
y_R &= j_R A_R^T e
\end{aligned}
$$

for given boundary conditions $e_G = 0$ and given resistance r, capacitance c and inductance l as well as prescribed currents $\bar{j}_I$ and prescribed voltages $\bar{v}_V$ and $\bar{v}_B$. The coupling variables are expressed as temperature of the resistor u_R, the temperature of the capacitor u_C, the temperature of the battery u_B and the energy flux of the resistor y_R.

Solid Network

We consider a solid network $\mathscr{N}_S = \{SW, LW, HT, HS, TB\}$ that is composed of solid walls SW, lumped walls LW, heat transfers HT, heat sources HS and temperature boundaries TB. The DAE for the network $\mathscr{N}_S$ in input-output form is given by: For given continuous inputs $(u_{Hs_S}^T, u_{Tb_S}^T)^T$, find the temperatures $(T_{Sw}^T, T_{Lw}^T)^T$, the heat fluxes $(H_{Ht_S}^T)^T$ and the outputs $(y_{Sw}^T, y_{Lw}^T, y_{Ht_S}^T)^T$, such that

$$
\begin{aligned}
m_{Sw} c_{p,Sw} \frac{dT_{Sw}}{dt} &= A_{Sw,Ht_S} H_{Ht_S} + A_{Sw,Hs} H_{Hs} + A_{Sw,Hs_u} u_{Hs_S}\\
0 &= A_{Lw,Ht_S} H_{Ht_S} + A_{Lw,Hs} H_{Hs} + A_{Lw,Hs_u} u_{Hs_S}\\
H_{Ht_S} &= c_{Ht_S} \left(A_{Sw,Ht_S}^T T_{Sw} + A_{Lw,Ht_S}^T T_{Lw} + A_{Tb,Ht_S}^T T_{Tb} + A_{Tb_u,Ht_S}^T u_{Tb_S} \right)\\
y_{Sw} &= |(A_{Sw,Hs_u}^T + A_{Tb_u,Ht_S} A_{Sw,Ht_S}^T)| T_{Sw}\\
y_{Lw} &= |(A_{Lw,Hs_u}^T + A_{Tb_u,Ht_S} A_{Lw,Ht_S}^T)| T_{Lw}\\
y_{Ht_S} &= A_{Tb_u,Ht_S} H_{Ht_S}
\end{aligned}
$$

for given boundary conditions $H_{Hs} = \bar{H}_{Hs}$ and $T_{Tb} = \bar{T}_{Tb}$ and given positive definite coefficient matrices m_{Sw}, $c_{p,Sw}$ and c_{Ht_S}. The coupling variables are expressed as the energy fluxes u_{Hs_S} and u_{Tb_S} and the temperatures y_{Sw}, y_{Lw} and y_{Ht_S}.

Fluid Network

We consider a fluid network $\mathscr{N}_F = \{PI, PU, DE, VJ, LJ, RE, HT, TB\}$ that is composed of pipes PI, pumps PU, demands DE, volume junctions VJ, lumped junctions LJ, reservoirs RE, heat transfers HT and temperature boundaries TB. The DAE for the network $\mathscr{N}_F$ in input-output form is given by: For given continuous inputs $(u_{Hs_F}^T, u_{Tb_F}^T)^T$, find the pressures $(p_{Lj}^T, p_{Vj}^T)^T$ the mass flows $(q_{Pi}^T, q_{Pu}^T)^T$, the temperatures $(T_{Vj}^T, T_{Lj}^T)^T$, the heat fluxes $(H_{Ht_F}^T, H_{Pu}^T, H_{Pi}^T)^T$ and the outputs $(y_{Vj}^T, y_{Lj}^T, y_{Ht_F}^T)^T$, such that

$$
\begin{aligned}
\frac{dq_{Pi}}{dt} &= c_{1,Pi}\left(A_{Jc,Pi}^T p_{Jc} + A_{Re,Pi}^T p_{Re}\right) + c_{2,Pi}\mathrm{diag}\left(|\mathrm{q_{Pi}}|\right)\mathrm{q_{Pi}} + \mathrm{c_{3,Pi}} \\
f_{Pu}(q_{Pu}) &= A_{Jc,Pu}^T p_{Jc} + A_{Re,Pu}^T p_{Re} \\
0 &= A_{Jc,Pi} q_{Pi} + A_{Jc,Pu} q_{Pu} + A_{Jc,De} q_{De} \\
m_{Vj} c_{p,Vj} \frac{dT_{Vj}}{dt} &= A_{Vj,Pi} H_{Pi} + A_{Vj,Pu} H_{Pu} + A_{Vj,De} H_{De} + A_{Vj,Ht_F} H_{Ht_F} + A_{Vj,Hs_u} u_{Hs_F} \\
0 &= A_{Lj,Pi} H_{Pi} + A_{Lj,Pu} H_{Pu} + A_{Lj,De} H_{De} + A_{Lj,Ht_F} H_{Ht_F} + A_{Lj,Hs_u} u_{Hs_F} \\
H_{Pi} &= B_{Jc}(q_{Pi}) T_{Vj} + B_{Jc}(q_{Pi}) T_{Lj} + B_{Jc}(q_{Pi}) T_{Re} \\
H_{Pu} &= B_{Jc}(q_{Pu}) T_{Vj} + B_{Jc}(q_{Pu}) T_{Lj} + B_{Jc}(q_{Pu}) T_{Re} \\
H_{Ht_F} &= c_{Ht_F}\left(A_{Vj,Ht_F}^T T_{Vj} + A_{Lj,Ht_F}^T T_{Lj} + A_{Tb_u,Ht_F}^T u_{Tb_F}\right) \\
y_{Vj} &= |(A_{Vj,Hs_u}^T + A_{Tb_u,Ht_F} A_{Vj,Ht_F}^T)| T_{Vj} \\
y_{Lj} &= |(A_{Lj,Hs_u}^T + A_{Tb_u,Ht_F} A_{Lj,Ht_F}^T)| T_{Lj} \\
y_{Ht_F} &= (A_{Tb_u,Ht_F} + A_{Lj,Hs_u}^T A_{Lj,Ht_F}^T + A_{Vj,Hs_u}^T A_{Vj,Ht_F}^T) H_{Ht_F}
\end{aligned}
$$

for given boundary conditions $q_{De} = \bar{q}_{De}$, $H_{De} = \bar{H}_{De}$, $p_{Re} = \bar{p}_{Re}$ and $T_{Re} = \bar{T}_{Re}$ and given coefficients $c_{1,Pi}$, $c_{2,Pi}$, $c_{1,Pi}$, m_{Vj}, $c_{p,Vj}$ and c_{Ht_F} as well as given functions f_{Pu}. The function B_{Jc} checks for the sign of the mass flow q_{Pi}, cf. [4]. The coupling variables are expressed as the temperatures u_{Hs_F} and u_{Tb_F} and the energy fluxes y_{Vj}, y_{Lj} and y_{Ht_F}.

Multi-Physical Model

To set up a mathematical model the characteristic equations of the individual network elements are combined with the actual network graph, yielding a DAE: Find x, $\dot{x}$, such that

$$F(\dot{x}, x, t) = 0. \tag{1}$$

DAEs resulting from automated modeling software typically obtain a structure with (differential) index greater 1, cf. [4–6] and hence are not suitable for a direct simulation with standard solvers. In the setup of multiple physical networks it is not sufficient, that the full DAE (1) can be reduced to a (differential) index 1. Additionally, each subsystem, to which a solver is applied, has to fulfill (differential) index 1 conditions as well, cf. [2]. In our applications an automatic index reduction is performed if the electric or the fluid system happens to be of (differential) index 2.

3 Multirate Integration for Coupled Network DAEs

The full DAE is partitioned due to the physical background to n subsystems (typically $n \gg 2$). Each subsystem is index reduced according to the available literature, cf. [4–6]. This approach yields a coupled system of n semi-explicit DAEs in input-output form of (differential) index 1. For given inputs u_i, find x_i, $\dot{x}_i$, a_i and y_i, such that

$$\begin{aligned} \dot{x}_i &= f_i(x_i, a_i, u_i, t) \\ 0 &= r_i(x_i, a_i, u_i, t) \\ y_i &= g_i(x_i, a_i, u_i, t) \end{aligned} \tag{2}$$

for $i = 1, \ldots, n$. The interaction of the individual systems is described via the input-output coupling $u_i = C_{ij} y_j$ and represents the electro-thermal coupling of the electric network and the cooling systems. A careful choice of the connectivity matrix C_{ij} guarantees that the coupled system obtains (differential) index 1 as well, cf. [1]. For each subsystem (2) an arbitrary Runge-Kutta method with micro-step sizes h_i is used, cf. Figure 1. The choice of the actual integration technique depends on the properties of the underlying system and can be explicit, implicit, fixed or adaptive. The whole system (1) is integrated via a non-iterative co-simulation technique with macro-step size $H = \max(h_i)$. All systems are updated at the end of each macro-step. This principle relates to synchronous communication and we refer to these points in time as synchronization times, cf. Figure 1. The evaluation of each macro-step of the subsystems is done in a sequential Gauss-Seidel-approach. The values u_i are handled with appropriate interpolation or extrapolation techniques, depending on the slow or active characteristic of the interacting subsystems. Due to $n \gg 2$ *slow-first* or *fast-first* strategies (cf. [3]) have been extended to strategies, that can be used for an arbitrary number n of components.

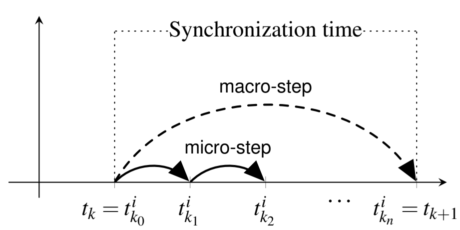

Fig. 1: Macro-step of the i-th system from synchronization time t_k to t_{k+1}.

4 Simulation of a BEV with Cooling System

We consider a BEV that demonstrates the modeling of an electrical system coupled to the required cooling system, cf. Figure 2. The model consists of an electrical propulsion and two cooling circuits. An oil circuit is used for cooling of the electric machine and a coolant circuit is used for cooling of the battery pack, inverter and low voltage DC-DC converter. The involved subsystem of the coupled electro-thermal model can be reduced to DAEs of (differential) index 1. The multirate approach pre-

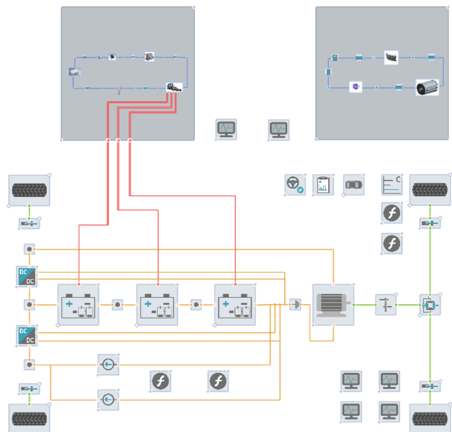

Fig. 2: Schematic representation of a BEV with cooling system in AVL CRUISE$^{\text{TM}}$M.

sented in Section 3 is put into context with the reference solution of a single solver approach (both sequential/single CPU). In this example eight thermal circuits, three mechanic circuits, an electric circuit, 14 gas circuits and two fluid circuits are present which represent in total 461 equations. The solvers for both, the single solver approach and the multirate approach are all adaptive explicit solvers [8]. Hence the step size of the single solver is limited to the minimum step size of all subdomains, while the multirate approach is limited to the synchronization time or to the characteristic of its own domain. Here the synchronization times are after each macro-step of 20*ms*. The simulation time of a singlerate case (in red) is compared with those of a multirate case (in blue) using AVL CRUISE$^{\text{TM}}$ M, cf. Figure 3. A significant speed up in the calculation time can be achieved, while the accuracy of the solution is still sufficiently high due to the adaptivity of the individual solvers.

5 Simulation of a Three Phase Inverter with Cooling System

We consider a detailed physical model of an inverter with switches/transistors, an RC (resistor-capacitor) filter as well as a 3 phase ohmic load. The inverter is used

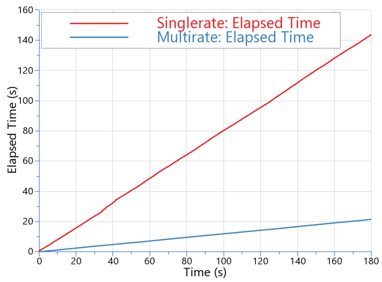

Fig. 3: Comparison of elapsed time of a multirate case against a single solver case for BEV with cooling system simulation.

Table 1: Comparison of singlerate and multirate approach corresponding CPU-time and average real time factor (RTF) for BEV with cooling system simulation.

Case	CPU-Time	Avg RTF
Singlerate	144.98	0.805447
Multirate	21.03	0.116853

to convert a DC (direct current) voltage through timed switching of the six transistors into a PWM (pulse width modulation) signal. The RC filter then averages the PWM and thus creates a 3 phase AC (alternating current) voltage, cf. Figure 4. In total this example consists of 178 equations which are spread over 20 solvers.

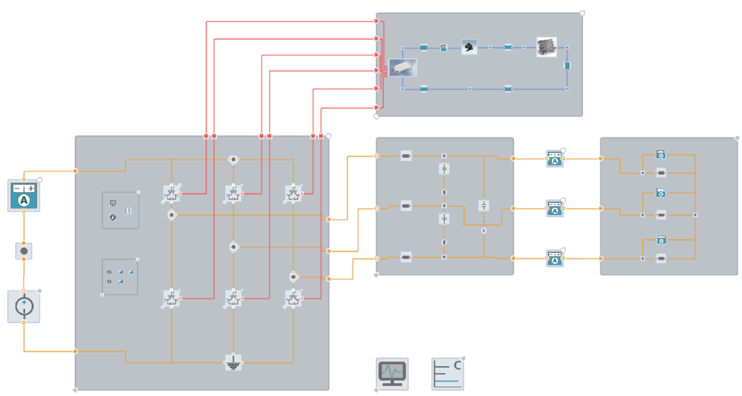

Fig. 4: Schematic representation of a three-phase inverter with cooling system in AVL CRUISE$^{\text{TM}}$M.

A fluid circuit, seven gas circuits and eleven thermal circuits are responsible for modeling the cooling. In the multirate scheme each circuit is solved individually with one scheme. For all of them an explicit fixed step method with a step size of $1ms$ is used. On the other hand the electric network is solved by its own scheme as well. Again an explicit fixed step method is used, whereby the chosen step size is now $1\mu s$. The information exchange takes place after each macro-step of $1ms$. This model is of special interest, since the electric network and the fluid network run on completely different time scales (of order $\mathscr{O}(1000)$). Again significant speed up in the calculation time can be achieved.

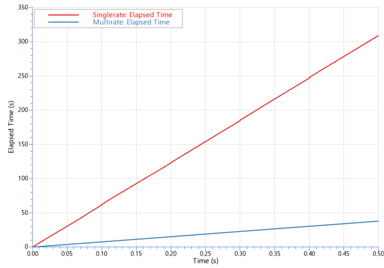

Fig. 5: Comparison of elapsed time of a multirate case against a single solver case for three-phase inverter with cooling system simulation.

Table 2: Comparison of singlerate and multirate approach corresponding CPU-time and average real time factor for the three-phase inverter with cooling system simulation.

Case	CPU-Time	Avg RTF
Singlerate	303.38	606.76734
Multirate	37.37	74.73045

6 Conclusion

As shown, the multirate approach offers a possibility to reduce computation time considerably. In order to ensure a stable simulation, automatic index reduction of the physical networks, appropriate solver settings for each subsystem and an adequate coupling procedure, play a decisive role. For the correct choice a significant speed up can be achieved, while conserving the accuracy criteria.

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